

Answer on Question #51644 – Math – Differential Calculus | Equations

Question

Differentiate the following functions

$$y = ((6x)^3 + (4x)^2 + 3)((2x)^2 - (4x)^{-2} + 5)$$

Solution

Method 1

$$y = (((6x)^3 + (4x)^2 + 3)((2x)^2 - (4x)^{-2} + 5));$$

Let $u = (6x)^3 + (4x)^2 + 3$, and $v = (2x)^2 - (4x)^{-2} + 5$, then

$$u' = ((6x)^3 + (4x)^2 + 3)' = ((6x)^3)' + ((4x)^2)' + (3)' = 3(6x)^2 \cdot 6 + 2(4x) \cdot 4 + 0 \\ = 18(6x)^2 + 8(4x)$$

$$v' = ((2x)^2 - (4x)^{-2} + 5)' = ((2x)^2)' + (-(4x)^{-2})' + (5)' \\ = 2(2x) \cdot 2 + (-)(-2)(4x)^{-3} \cdot 4 + 0 = 4(2x) + 8(4x)^{-3}$$

$$y' = (uv)' = u'v + uv' = [18(6x)^2 + 8(4x)] \cdot [(2x)^2 - (4x)^{-2} + 5] + [(6x)^3 + (4x)^2 + 3] \cdot [4(2x) + 8(4x)^{-3}] = 4320x^4 + 256x^3 + \frac{3}{8x^3} + 3240x^2 + 184x - \frac{27}{2};$$

Method 2

$$y' = (((6x)^3 + (4x)^2 + 3) \cdot [(2x)^2 - (4x)^{-2} + 5])' \\ = (((6x)^3(2x)^2 - (6x)^3(4x)^{-2} + 5(6x)^3 + (4x)^2(2x)^2 - (4x)^2(4x)^{-2} \\ + 5(4x)^2 + 3(2x)^2 - 3(4x)^{-2} + 15))' =$$

$$= (6^3 2^2 x^5 - 6^3 4^{-2} x + 5 \cdot 6^3 x^3 + 4^2 2^2 x^4 - 1 + 5 \cdot 4^2 x^2 + 3 \cdot 2^2 x^2 - 3 \cdot 4^{-2} x^{-2} + 15)' \\ = 5 \cdot 6^3 2^2 x^4 - 6^3 4^{-2} + 5 \cdot 6^3 \cdot 3x^2 + 4^2 2^2 \cdot 4x^3 + 5 \cdot 4^2 \cdot 2x + 3 \cdot 2^2 \cdot 2x - 3 \cdot 4^{-2} (-2)x^{-3} = \\ = 4320x^4 + 256x^3 + 3240x^2 + 184x - \frac{27}{2} + \frac{3}{8x^3}$$

Answer: $y' = 4320x^4 + 256x^3 + \frac{3}{8x^3} + 3240x^2 + 184x - \frac{27}{2}$