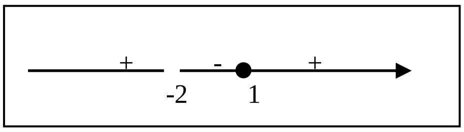
Answer on Question #51627 - Math - Real analysis

What is the domain and range of $f(x)=\sqrt{\dfrac{x-1}{x+2}}$?

Solution:

Let's find domain of $f(x)$:

$$\frac{x-1}{x+2}\geq 0$$



Thus, the domain of $f(x)$ is $D(f)=(-\infty,-2)\cup[1,+\infty)$.

Let's find the derivative of $f(x)$:

$$f'(x)=\frac{1}{2}\sqrt{\frac{x+2}{x-1}}\,\frac{x+2-x+1}{(x+2)^2}=\sqrt{\frac{x+2}{x-1}}\,\frac{3}{2(x+2)^2}\geq 0 \quad \text{on the domain} \quad D(f)=(-\infty,-2)\cup[1,\infty).$$

Therefore $f(x)$ is monotonic increasing on $(-\infty,-2)$ and $f(x)$ is monotonic increasing on $(1,\infty)$.

Let's find limits:

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\sqrt{\frac{x-1}{x+2}}=\lim_{x\to-\infty}\sqrt{\frac{x}{x}\cdot\frac{1-\frac{1}{x}}{1+\frac{2}{x}}}=1;$$

$$\lim_{x\to-2-0} f(x)=\lim_{x\to-2-0}\sqrt{\frac{x-1}{x+2}}=\left\{\sqrt{\frac{-3}{-0}}\right\}=+\infty;$$

$$f(1)=\sqrt{\frac{1-1}{1+2}}=0;$$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\sqrt{\frac{x-1}{x+2}}=\lim_{x\to+\infty}\sqrt{\frac{x}{x}\cdot\frac{1-\frac{1}{x}}{1+\frac{2}{x}}}=1.$$

Besides, $f(x)=\sqrt{\dfrac{x-1}{x+2}}=\sqrt{\dfrac{x+2-3}{x+2}}=\sqrt{1-\dfrac{3}{x+2}}\neq 1$ for every x.

Since, $f(x)$ is continuous function on the domain $D(f)=(-\infty,-2)\cup[1,+\infty)$, then we obtain the range of $f(x)$: $E(f)=[0,1)\cup(1,\infty)$.

Answer: $D(f)=(-\infty,-2)\cup[1,\infty)$, $E(f)=[0,1)\cup(1,+\infty)$.