Answer on Question #51527 – Math – Statistics and Probability

Question

A sample of 64 students from a large university is taken. The average age in the sample was 22 years with a standard deviation of 6 years. Construct a 95% confidence interval for the average age of the population.

Solution

$n = 64$

$\bar{X} = 22$

$\sigma = 6$

$\gamma = 95\% \Rightarrow \alpha = 5\% = 0.05$

$$S^2 = \frac{n}{n-1}\sigma^2$$

$$S = \sqrt{\frac{n}{n-1}}\sigma = \sqrt{\frac{64}{63}}6 = 6.05$$

$$P\left(\bar{X} - t_{1-\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{X} + t_{1-\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}\right) = 1-\alpha$$

where $t_{1-\frac{\alpha}{2},n-1}$ - $1-\frac{\alpha}{2}$ -level quantile of Student's distribution with $n-1$ degrees of freedom. $t_{1-\frac{0.05}{2},100-1} = 1.984$.

Then

$t_{1-\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}} = 1.984\frac{6.05}{\sqrt{64}} = 1.5$, so

$22 - 1.5 \le \mu \le 22 + 1.5$

$20.5 \le \mu \le 23.5$

is 95% confidence interval for the average age of the population.