Answer on Question #51454 – Math – Statistics and Probability

Question

Police estimate that 80% of drivers wear their seatbelts. They set up a safety road block stopping, cars to check for seatbelt use. If they stop 9 cars, what is the probability that:

1.1) Exactly three drivers are not wearing their seatbelts?

1.2) At least 3 drivers are not wearing their seatbelt?

1.3) At least 2 drivers but no more than 4 drivers are not wearing their seatbelt?

Solution

If police estimate that 80% of drivers wear their seatbelts, then we can consider that 20% of drivers don't wear their seatbelts. Then probability:

$$P(\text{"}driver\ is\ not\ wearing\ seatbelt\text{"}) = 0.2$$

Let X be a number of drivers who are not wearing seatbelt. Then X - the number of successes in 9 independent experiments. Then using binomial distribution,

$$P(X=k) = C_9^k p^k (1-p)^{9-k}$$

where $p = 0.2,\ C_9^k = \dfrac{9!}{k!(9-k)!}$

1.1)

$$P(X=3) = C_9^3 p^3 (1-p)^6 = \frac{9!}{3!6!} 0.2^3 0.8^6 = \frac{7\cdot 8\cdot 9}{6} 0.008\cdot 0.262144 = 0.176160768$$

1.2) "At least 3 drivers are not wearing their seatbelt" = "3 or 4, or 5, … or 9 drivers are not wearing their seatbelt "

$$P(X=3)+P(X=4)+\ldots+P(X=9) = 1-P(X=0)-P(X=1)-P(X=2) =$$
$$= 1 - C_9^0 p^0 (1-p)^9 - C_9^1 p^1 (1-p)^8 - C_9^2 p^2 (1-p)^7 = 1 - 0.8^9 - 9\cdot 0.2\cdot 0.8^8 -$$
$$-\frac{8\cdot 9}{2}\cdot 0.2^2\cdot 0.8^7 = 1 - 0.134217728 - 0.301989888 - 0.301989888 = 0.261800224$$

1.3) "At least 2 drivers but no more than 4 drivers are not wearing their seatbelt?" = "2 or 3, or 4 drivers are not wearing their seatbelt "

$$P(X=2)+P(X=3)+P(X=4)=C_9^2p^2(1-p)^7+C_9^3p^3(1-p)^6+C_9^4p^4(1-p)^5=$$
$$=p^2(1-p)^5\left(C_9^2(1-p)^2+C_9^3p(1-p)+C_9^4p^2\right)=0.2^20,8^5\left(23.04+13.44+5.04\right)=$$
$$=0.544210944$$