Answer on Question #51428 - Math - Real Analysis

1) Test the following series for convergence:

(a) $\sum_{n=1}^{\infty} n^{1/n}$

Solution

Let us check the vanishing condition:

$\lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} e^{\frac{\ln n}{n}} = e^0 = 1 \neq 0$, Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty} n^{1/n}$ diverges.

Answer: the series diverges.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin 3nx}{n^3}$

Solution

Let test it for absolute convergence:

$\sum_{n=1}^{\infty} \left|\frac{(-1)^n \sin 3nx}{n^3}\right| = \sum_{n=1}^{\infty} \frac{|\sin 3nx|}{n^3}$, apply the comparison test $0 \le \frac{|\sin 3nx|}{n^3} \le \frac{1}{n^3}$, but the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges because the power of n in denominator is greater than 1. So, $\sum_{n=1}^{\infty} \frac{(-1)^n \sin 3nx}{n^3}$ is absolutely convergent for every real x.

Answer: the series is absolutely convergent for any $x \in R$.