Answer on Question #51427 – Math – Real Analysis

Show that:

$$\sum_{n=0}^{\infty} \frac{1}{(a+n)*(a+n+1)} = \frac{1}{a} \qquad for\ \ a > 0;$$

Solution

$$\frac{1}{(a+n)*(a+n+1)} = \frac{1+a+n-a-n}{(a+n)*(a+n+1)} = \frac{(1+a+n)-(a+n)}{(a+n)*(a+n+1)} =$$

$$= \frac{1+a+n}{(a+n)*(a+n+1)} - \frac{a+n}{(a+n)*(a+n+1)} = \frac{1}{a+n} - \frac{1}{a+n+1};$$

Let

$$S_N = \sum_{n=0}^{N} \frac{1}{(a+n)*(a+n+1)} = \sum_{n=0}^{N} \left(\frac{1}{a+n} - \frac{1}{a+n+1}\right) = \left(\frac{1}{a} - \frac{1}{a+1}\right) + \left(\frac{1}{a+1} - \frac{1}{a+2}\right) + \cdots +$$
$$+ \left(\frac{1}{a+N} - \frac{1}{a+N+1}\right) = \frac{1}{a} - \frac{1}{a+1} + \frac{1}{a+1} - \frac{1}{a+2} \ldots + \frac{1}{a+N} - \frac{1}{a+N+1} = \frac{1}{a} - \frac{1}{a+N+1};$$

$$S = \sum_{n=0}^{\infty} \frac{1}{(a+n)(a+n+1)} = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \left(\frac{1}{a} - \frac{1}{a+N+1}\right) = \frac{1}{a} - 0 = \frac{1}{a}$$

Answer: $1/a$.