Answer on Question #51409 – Math – Trigonometry

$4\sin^2(x) + 7\cos(x) = 6$

Solution

Pythagorean trigonometric identity

$$\sin^2(x) + \cos^2(x) = 1$$

gives

$$\sin^2(x) = 1 - \cos^2(x).$$

Substitute for the initial equation

$$4\sin^2(x) + 7\cos(x) = 6$$

and obtain

$4(1 - \cos^2(x)) + 7\cos(x) = 6$, open brackets $4 - 4\cos^2(x) + 7\cos(x) = 6$;

collect similar terms

$$4\cos^2(x) - 7\cos(x) + 2 = 0.$$

Make a substitution

$$\cos(x) = t$$

such that $-1 \le t \le 1$.

After substitution, equation becomes as follows

$$4t^2 - 7t + 2 = 0.$$

To solve it, calculate

$D = 7^2 - 4 \cdot 4 \cdot 2 = 49 - 32 = 17;$

$$\left[\begin{array}{l} t_1 = \frac{7-\sqrt{17}}{2 \cdot 4} = \frac{7-\sqrt{17}}{8} < \frac{7-\sqrt{16}}{8} = \frac{7-4}{8} = \frac{3}{8} < 1 \\ t_2 = \frac{7+\sqrt{17}}{2 \cdot 4} = \frac{7+\sqrt{17}}{8} > \frac{7+\sqrt{16}}{8} = \frac{7+4}{8} = \frac{11}{8} > \frac{8}{8} = 1 \end{array}\right.$$

Because $t_2 = \frac{7+\sqrt{17}}{8} > 1$ does not satisfy the condition $-1 \le t \le 1$, skip this value, equation $\cos(x) = t_2$ has no roots.

Because $t_1 = \frac{7-\sqrt{17}}{8} < 1$ satisfies the condition $-1 \le t \le 1$, equation $\cos(x) = t_1$ has roots.

We have

$$\cos(x) = \frac{7-\sqrt{17}}{8} \;\Rightarrow\; x = \cos^{-1}\left(\frac{7-\sqrt{17}}{8}\right) + 2\pi n,$$

where $\cos^{-1}(x)$ is the inverse cosine function, n is integer.

Answer: $x = 2\pi n + \cos^{-1}\left(\frac{7-\sqrt{17}}{8}\right),\ n \in \mathbb{Z}.$