

### Answer on Question #51399 – Math – Statistics and Probability

A departmental store has a single cashier. During the rush hours, customers arrive at the rate of 15 customers per hour. The average number of customers that can be processed by the cashier is 18 per hour. Assume that the conditions for the use of single-channel queuing model apply. What is the:

- a) Probability that the cashier is idle?
- b) Average number of customers in the queuing system?
- c) Average time a customer spends in the system?
- d) Average number of customers in the queue?
- e) Average time a customer spends in the queue waiting for service?

#### Solution

Mean arrival rate  $a = 15$  customers per hour

Mean service rate  $s = 18$  customers per hour.

The problem deals with M/M/1 queue.

- a) Probability that the cashier is idle (there are 0 customers in the system)

$$p_0 = 1 - \frac{a}{s} = 1 - \frac{15}{18} = \frac{1}{6};$$

- b) Average number of customers in the queuing system

$$N = \frac{a}{s-a} = \frac{15}{3} = 5;$$

- c) Average time a customer spends in the system

$$W = \frac{N}{a} = \frac{1}{s-a} = \frac{1}{3} \text{ hour} = 20 \text{ min}$$

- d) Average number of customers in the queue

$$N_q = aW_q = \frac{a^2}{s(s-a)} = \frac{225}{18 \cdot 3} \approx 4.17;$$

e) Average time a customer spends in the queue

$$W_q = W - \frac{1}{s} = \frac{1}{s-a} - \frac{1}{s} = \frac{a}{s(s-a)} = \frac{5}{18} \text{ hour} \approx 16.7 \text{ min.}$$