

Answer on Question #51352 – Math – Geometry

Grains of fine California beach sand are approximately spheres with an average radius of $50\ \mu\text{m}$ and are made of silicon dioxide, which has a density of $2.4 \times 10^3\ \text{kg/m}^3$. What mass of sand grains would have a total surface area (the total area of all the individual spheres) equal to the surface area of cube $1.2\ \text{m}$ on an edge.

Solution

$$r = 50\ \mu\text{m} = 5 \times 10^{-5}\ \text{m}; \quad \rho = 2.4 \times 10^3\ \text{kg/m}^3; \quad a = 1.2\ \text{m}; \quad m = ?$$

One grain has the surface area: $S_g = 4\pi r^2$.

The surface area of cube: $S_c = 6a^2$.

Number of grains, that have total surface area equal to the surface area of cube, is given by

$$N = S_c / S_g = \frac{3a^2}{2\pi r^2}.$$

Volume of one grain: $V_0 = \frac{4\pi r^3}{3}$.

$$m = \rho \cdot V_0 \cdot N = \rho \frac{4\pi r^3 \cdot 3a^2}{3 \cdot 2\pi r^2} = 2\rho r a^2 = 2 \cdot 2.4 \cdot 10^3 \cdot 5 \cdot 10^{-5} \cdot 1.44 = \mathbf{0.3456\ \text{kg}}$$

Answer: $m = 0.3456\ \text{kg}$