# Answer on Question #51347 - Math – Analytic Geometry

The orthocentre of the triangle formed by the lines x-2y+9=0,x+y-9=0,2x-y-9=0 is ?

**Solution**

Let's find the coordinates of vertices $A, B, C$ of triangle:

$$\begin{cases} x-2y+9=0 \\ x+y-9=0 \end{cases} \Rightarrow \begin{cases} x-2y+9=0 \\ 3y-18=0 \end{cases} \Rightarrow \begin{cases} x-12+9=0 \\ y=6 \end{cases} \Rightarrow \begin{cases} x=3 \\ y=6 \end{cases}. \text{ So, } A(3,6)$$

$$\begin{cases} x-2y+9=0 \\ 2x-y-9=0 \end{cases} \Rightarrow \begin{cases} x-2y+9=0 \\ y=2x-9 \end{cases} \Rightarrow \begin{cases} x-2(2x-9)+9=0 \\ y=2x-9 \end{cases} \Rightarrow \begin{cases} -3x+27=0 \\ y=2x-9 \end{cases} \Rightarrow \begin{cases} x=9 \\ y=9 \end{cases}. \text{ So, } B(9,9)$$

$$\begin{cases} x+y-9=0 \\ 2x-y-9=0 \end{cases} \Rightarrow \begin{cases} x+y-9=0 \\ 3x-18=0 \end{cases} \Rightarrow \begin{cases} 6+y-9=0 \\ x=6 \end{cases} \Rightarrow \begin{cases} y=3 \\ x=6 \end{cases}. \text{ So, } C(6,3)$$

Assume that $O(x, y)$ is a orthocenter of the triangle, then $AO \perp BC$ and $BO \perp AC$, which give that the dot product of vectors $\overline{AO}$ and $\overline{BC}$ is zero, the dot product of vectors $\overline{BO}$ and $\overline{AC}$ is zero. From $\overline{AO} = (x-3, y-6)$, $\overline{BC} = (-3,-6)$, $\overline{BO} = (x-9, y-9)$, $\overline{AC} = (3,-3)$ we obtain:

$$\begin{cases} \overline{AO} \cdot \overline{BC} = 0 \\ \overline{BO} \cdot \overline{AC} = 0 \end{cases} \Rightarrow \begin{cases} 9-3x-6y+36=0 \\ 3x-27-3y+27=0 \end{cases} \Rightarrow \begin{cases} 9y=45 \\ x=y \end{cases} \Rightarrow \begin{cases} y=5 \\ x=5 \end{cases}$$

**Answer:** orthocenter of the triangle is $O(5,5)$.