

Answer on Question #51346 – Math – Analytic Geometry

Task

the orthocentre of the triangle formed by the lines $x+y+1=0$, $x-y-1=0$, $3x+4y+5=0$ is

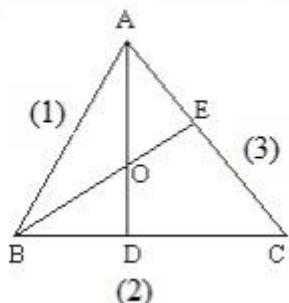
Solution

Introduce equations of sides of the triangle

$$AB: x+y+1=0 \quad (1)$$

$$BC: x-y-1=0 \quad (2)$$

$$AC: 3x+4y+5=0 \quad (3)$$



Equation of AB is $y = -x - 1$, its slope is $k_1 = -1$.

Equation of BC is $y = x - 1$, its slope is $k_2 = 1$.

Note that $k_1 \cdot k_2 = -1$, it means that AB and BC are perpendicular, hence, triangle ABC is right.

In right triangle ABC, the orthocenter is the polygon vertex B of the right angle.

Solving (1) and (2):

$$\begin{cases} x + y + 1 = 0 \\ x - y - 1 = 0 \end{cases} \Rightarrow \begin{cases} (x + y + 1) + (x - y - 1) = 0 \\ x - 1 = y \end{cases} \Rightarrow \begin{cases} 2x = 0 \\ y = x - 1 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = -1 \end{cases} \Rightarrow B(0; -1)$$

Answer: the orthocenter of the triangle is $(0; -1)$.