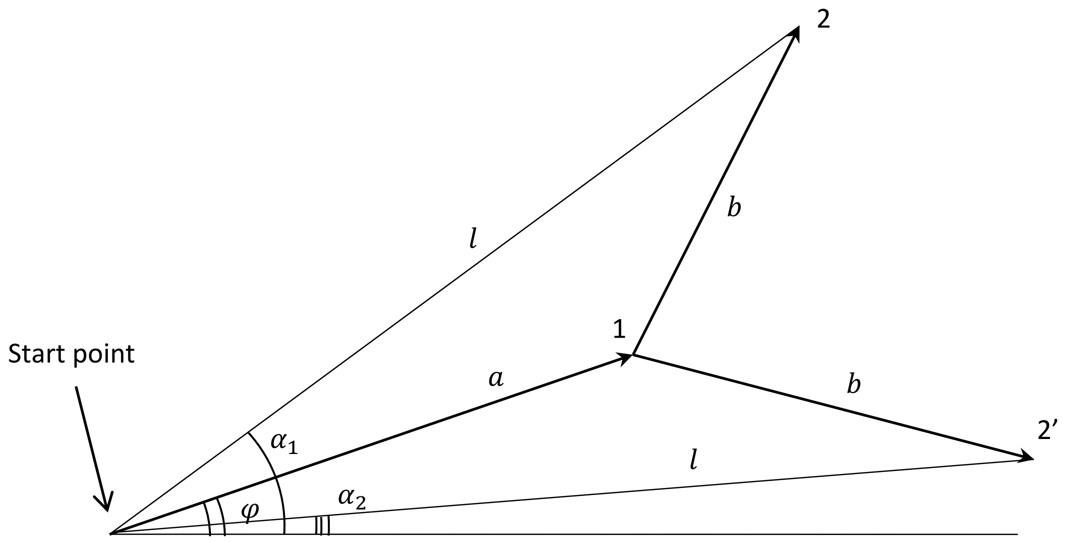
Answer on Question #51337 – Math – Vector Calculus

In an orienteering race the first checkpoint is $a = 500\text{m}$ from the start and on a bearing of $\varphi = 30°$. The second checkpoint is $b = 400\text{m}$ from the first checkpoint and $l = 600\text{m}$ from the start. Find, to the nearest degree, the two possible bearings α_1 *and* α_2 of checkpoint 2 from the start.

Solution



α_1:

According to the law of cosines used in triangle 1O2, where 1 is the first checkpoint, O is the start, 2 is one of possible second checkpoints, obtain

$$b^2 = a^2 + l^2 - 2al\cos(\alpha_1 - \varphi)$$

$$\alpha_1 = \arccos\left(\frac{a^2 + l^2 - b^2}{2al}\right) + \varphi = \arccos\left(\frac{250000\text{m}^2 + 360000\text{m}^2 - 160000\text{m}^2}{600000\text{m}^2}\right) + 30° =$$

$$= \arccos 0.75 + 30° = 41° + 30° = 71°$$

where arccos(x) is the inverse of cosine function cos(x).

α_2:

According to the law of cosines used in triangle 1O2$'$, where 1 is the first checkpoint, O is the start, 2$'$ is one of possible second checkpoints, obtain

$$b^2 = a^2 + l^2 - 2al\cos(\varphi - \alpha_2)$$

$$\alpha_2 = \varphi - \arccos\left(\frac{a^2 + l^2 - b^2}{2al}\right) = 30° - \arccos\left(\frac{250000\text{m}^2 + 360000\text{m}^2 - 160000\text{m}^2}{600000\text{m}^2}\right) =$$

$= 30° - \arccos 0.75 = 30° - 41° = -11°$ or $169°$

Answer: $\alpha_{1,2} = \varphi \pm \arccos\left(\frac{a^2+l^2-b^2}{2al}\right) = 71°, -11°.$