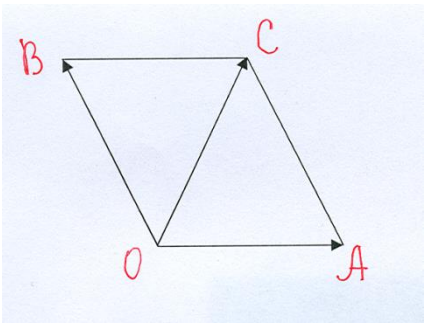
Answer on Question #51334 – Math – Vector Calculus

Two forces have magnitudes of 8N and 12 N and the angle between them is 130°. Find the magnitude of the resultant and the angle it makes with the larger of the two forces.



Figure

Solution

Step 1.

Let's use Parallelogram Law of Vector Addition: "If two vector quantities are represented by two adjacent sides of a parallelogram then the diagonal of the parallelogram will be equal to the resultant of these two vectors."

Let's denote $\angle BOA = 130^\circ$, $\left|\overrightarrow{OB}\right| = b = 12N$, $\left|\overrightarrow{OA}\right| = a = 8N$, $\left|\overrightarrow{OC}\right| = c$, $\angle BOC = \beta$.

Resulting force is $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OA}$, OBCA is a parallelogram, $\angle OBC = \angle OAC = 180^\circ - 130^\circ = 50^\circ$,
Let's consider triangle OBC. We find the magnitude of the resultant according to the Law of Cosines.

$$c^2 = b^2 + a^2 - 2ab\cos\angle OBC = 12^2 + 8^2 - 2\cdot 12\cdot 8\cos 50^\circ = 208 - 192\cos\frac{50\pi}{180} = 208 - 192\cdot 0.64 = 84.58$$

$$c = \sqrt{c^2} = \sqrt{84.58} = 9{,}2 \Rightarrow \left|\overrightarrow{OC}\right| = 9{,}2N$$

Step 2.

According to the Law of Sines, $\frac{BC}{\sin\angle BOC} = \frac{OC}{\sin\angle OBC}$, that is,

$$\frac{a}{\sin\beta} = \frac{c}{\sin 50^0} \Rightarrow \sin\beta = \frac{a}{c}\sin 50^\circ = \frac{8}{9{,}2}\sin 50^\circ = 0{,}67 \Rightarrow \beta = 41{,}8^\circ.$$

Answer: $\left|\overrightarrow{OC}\right| = 9{,}2N$, $\beta = 41{,}8^\circ$.