Answer on Question #51274 – Math – Trigonometry

Hexagon ABCDEF is inscribed in the circle with centre M and radius R. AB=CD=EF=R. Points I, J and K are the midpoints of segments BC, DE and FA respectively. Prove that triangle IJK is equilateral.

Note: Could you please use trigonometry? The method is to find MI and MJ, and then use cosine rule in triangle MIJ to find IJ. Then simplify the expression obtained for IJ, and as it will be symmetrical, the triangle will be equilateral.

Solution

It follows from the statement of question that hexagon ABCDEF is regular. Thus, all of its sides have the same length, which is equal to circumradius R, hence triangle *BMC* is equilateral, *BI=CI=BC/2=R/2.*

$$MI = \sqrt{BM^2 - BI^2} = \sqrt{R^2 - \frac{R^2}{4}} = \frac{R\sqrt{3}}{2} = MJ;$$

Using cosine rule in triangle *MIJ*,

$$IJ = \sqrt{MI^2 + MJ^2 - 2MI \cdot MJ \cos \angle IMJ} = \sqrt{\frac{3R^2}{4} + \frac{3R^2}{4} - 2\frac{3R^2}{4}\cos 120^\circ} = \frac{3}{2}R;$$

If we do similarly to find *JK* and *KI*, we obtain the same answer.

Finally, $IJ = JK = KI$, which means that triangle *IJK* is equilateral.