

Answer on Question #51195 - Math - Algebra

Question 1

$f: \mathbb{R} \setminus \{1/2\} \rightarrow \mathbb{R}$ $f(x) = \frac{x+3}{2x-1}$. IS IT INJECTIVE?

Solution

1) Definition:

$f: X \rightarrow Y$ is called injective if for any $x_1 \neq x_2$ in X , it follows that $f(x_1) \neq f(x_2)$.

2) $f(x) = \frac{x+3}{2x-1}$, $x \neq \frac{1}{2}$ given.

3) Take $x_1 \neq x_2$, $x_1, x_2 \in \mathbb{R} \setminus \{\frac{1}{2}\}$ and prove that $f(x_1) \neq f(x_2)$.

4) Suppose that $f(x_1) = f(x_2) \Rightarrow \frac{x_1+3}{2x_1-1} = \frac{x_2+3}{2x_2-1} \Rightarrow$

$$(x_1 + 3) \cdot (2x_2 - 1) = (x_2 + 3) \cdot (2x_1 - 1) \Rightarrow$$

$$2x_1 2x_2 - x_1 + 6x_2 - 3 = 2x_1 2x_2 - x_2 + 6x_1 - 3 \Rightarrow$$

$6x_2 + x_2 = 6x_1 + x_1 \Rightarrow 7x_1 = 7x_2 \Rightarrow x_1 = x_2$ that does not meet the condition $x_1 \neq x_2$, therefore that assumption is false.

This means that $f(x_1) \neq f(x_2)$ and by definition of injective function,

$$f(x) = \frac{x+3}{2x-1} \text{ is injective.}$$

Question 2:

IF $f(x_1)=f(x_2)$ and then $x_1=x_2$ it's injective.

so i want to check $f(1)=4$ $f(2)=5/3$. it never become same y value for different x value. but why to check is it injective or not we do $f(x_1)=f(x_2)$ and then $x_1=x_2$?

Solution

1) Definition:

$f: X \rightarrow Y$ is called injective if for any $x_1 \neq x_2$ in X , it follows that $f(x_1) \neq f(x_2)$.

2) $x_1 = 1, x_2 = 2, f(x_1)=4, f(x_2)=5/3$

3) For $x_1 \neq x_2$ matches $f(x_1) \neq f(x_2)$. As defined it's injective.

4) Equivalent mapping is an injection if

$$(f(x_1) = f(x_2)) \Rightarrow (x_1 = x_2)$$

Map is injective if and only if there exists for the left inverse $\exists g: Y \rightarrow X$ $g \circ f = id_X$,

where \circ denotes the composition and id_X the identity on X .