Answer on Question #51194 – Math – Statistics and Probability

Answer the following questions:

a. State the decision rule based on p-values for a two-sided hypothesis test at level $\alpha = 0.01$.

b. Consider the following hypothesis test where ($n = 100$).

$H0: \mu = 50$ $\alpha = 0.10$

$Ha: \mu > 50$

Suppose $z = -2.20$. Compute the p-value. Should H0 be rejected?

c. Consider the following hypothesis test where ($n = 100$).

$H0: \mu = 50$ $\alpha = 0.05$

$Ha: \mu < 50$

Suppose $z = -2.20$. Compute the p-value. Should H0 be rejected?

d. Consider the following hypothesis test where ($n = 100$).

$H0: \mu = 50$ $\alpha = 0.01$

$Ha: \mu \neq 50$

Suppose $z = -2.20$. Compute the p-value. Should H0 be rejected?

Solution

a. State the decision rule based on p-values for a two-sided hypothesis test at level $\alpha = 0.01$.

We have to start with the description of the rules.

The p-value is the probability of obtaining a sample value of the test statistic as extreme as the one we computed if the null hypothesis H_0 is true. P-values are inverse measures of the strength of evidence against the null hypothesis.

Small p-values or p-values close to zero - constitute strong evidence against the null hypothesis H_0. Large p-values or p-values close to one - provide only weak evidence against the null hypothesis.

If the p-value for the calculated sample value of the test statistic is less than the chosen significance level α, reject the null hypothesis at significance level α.

If we have p-value $< \alpha$ then we should to reject H_0 at significance level α.

If the p-value for the calculated sample value of the test statistic is greater than or equal to the chosen significance level α, retain (i.e., do not reject) the null hypothesis at significance level α.

If p-value $\geq \alpha$ we should to retain H_0 at significance level α.

Consider the P-Values for Two-Tail according to the condition of the task.

Null and Alternative Hypotheses

$H_0: \beta_2 = b_2$

$H_1: \beta_2 \neq b_2$ a two-sided alternative hypothesis.

Definition of two-tail p-value for t_0.

t_0 = the calculated sample value of the t-statistic for a given null hypothesis.

The two-tail p-value of t0 is the probability that the null distribution of the test statistic takes an absolute value greater than the absolute value of t_0, where the absolute value of t_0 is denoted as $|t_0|$. That is,

Two-tail p-value for $t_0 = P_r(|t| > |t_0|) = P_r(t > t_0) + Pr(t < -t_0) = 2\,Pr(t > t_0)$ if $t_0 > 0$

$= P_r(t < t_0) + Pr(t > -t_0) = 2\,Pr(t < t_0)$ if $t_0 < 0$

Two-tail p-value of t_0 is the probability of obtaining a t value greater in absolute size than the sample value t_0 if the null hypothesis $H_0: \beta_2 = b_2$ is in fact true.

When we use the Hypothesis Test we apply the following steps.

1. State the null and alternative hypotheses.

2. Collect and summarize the data so that a test statistic can be calculated. A test statistic is a single value summary of the data that has been standardized so that a p-value can be obtained.

3. Use the test statistic to find the p-value. The p-value represents the likelihood of getting our test statistic or any test statistic more extreme, if in fact the null hypothesis is true.

For a one-sided "greater than" alternative hypothesis, the "more extreme" part of the interpretation refers to test statistic values larger than the test statistic given.

For a one-sided "less than" alternative hypothesis, the "more extreme" part of the interpretation refers to test statistic values smaller than the test statistic given.

For a two-sided "not equal to" alternative hypothesis, the "more extreme" part of the interpretation refers to test statistic values that are "more extreme" than the given test statistic and are "more extreme" than the negative of the given test statistic (both "tails").

4. Make a decision using the p-value. State a conclusion in terms of the problem. If statistically significant, we can conclude the alternative hypothesis.

Decision Rule used with P-Values

If the p-value ≤ 0.01, we can conclude that there is a statistically significant result.

If the p-value > 0.01, we cannot conclude that there is a statistically significant result.

b. Consider the following hypothesis test where (n = 100).

H_0: μ = 50 α = 0.10

H_a: μ > 50

Suppose z = -2.20. Compute the p-value. Should H_0 be rejected?

From the table we find the p-value, which is equal to 0.0139. The result is significant at p < 0.10.

Since the P-value (0.0139) is less than the significance level (0.1), the null hypothesis should be rejected.

c. Consider the following hypothesis test where (n = 100).

H_0: μ = 50

H_a: μ < 50

α = 0.05

Suppose z = -2.20. Compute the p-value. Should H_0 be rejected?

From the table we can determine the value of p, which is equal to p-value, it is

0.013903.

The result is significant at p < 0.10.

Since the P-value (0.0139) is less than the significance level (0.05), the null hypothesis should be rejected.

d. Consider the following hypothesis test where (n = 100).

H_0: μ = 50

H_a: μ ≠ 50

α = 0.01

Suppose z = -2.20. Compute the p-value. Should H_0 be rejected?

Since it is a 2-tail test the p-value

$$= 2P(z < -2.2) = 0.0278$$

Thus, we can conclude that the result is not significant at $p < 0.01$. Since the p-value is greater than $\alpha = 0.01 = 1\%$, we fail to reject H_0.