Question #51178, Math, Real Analysis | for confirmation

Rolle's Theorem *was* named after the French mathematician Michel Rolle (1652–1719), who proposed the first known formal proof of it in 1691. The name "Rolle's theorem" was first appeared in 1834.
In calculus, **Rolle's theorem** essentially states that any real-valued differentiable function that attains equal values at two distinct points must have a stationary point somewhere between them — that is, a point where the first derivative (the slope of the tangent line to the graph of the function) is zero.
Rolle's Theorem *gives conditions that guarantee the existence of an extreme value in the interior of a closed interval.*
The Extreme Value Theorem states that a continuous function on a closed interval must have both a minimum and a maximum on the interval. Let's consider the case when both of these values can occur at the endpoints.
Rolle's Theorem
Let's consider a function f which is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.
Proof
Let $f(a)=q=f(b)$.
Case 1. If $f(x)=q$ for $\forall x$ in $[a,b]\Rightarrow f(x)$ is ***constant*** on the interval, then by ***the Constant Rule Theorem,*** which states ***that the derivative of a constant function is 0,*** $f'(x)=0$ for $\forall x$ in (a,b).
Case 2. Assume that $f(x)>q$ for some value of x in (a,b). Then by ***the Extreme Value Theorem*** $f(x)$ has a maximum at some c in the interval. Moreover, because $f(x)>q$, this maximum does not occur at either endpoints. Therefore, $f(x)$ has a maximum in the open interval (a,b). This implies that $f(c)$ is a relative maximum and, we can apply ***the Theorem about Relative Extrema*** which states that Relative Extrema occur only at critical numbers of $f(x)$. So, c is a critical number of $f(x)$. And because $f(x)$ ***is differentiable function at*** $c \Rightarrow f'(c)=0$.
Case 3. Assume that $f(x)<q$ for some value of x in (a,b). Then by ***the Extreme Value Theorem*** $f(x)$ has a minimum at some c in the interval. Moreover, because $f(x)<q$, this minimum does not occur at either endpoints. Therefore, $f(x)$ has a minimum in the open interval (a,b). This implies that $f(c)$ is a relative minimum and, we can apply ***the Theorem about Relative Extrema*** which states that Relative Extrema occur only at critical numbers of $f(x)$. So, c is a critical number of $f(x)$. And because $f(x)$ ***is differentiable function at*** $c \Rightarrow f'(c)=0$.
Geometric interpretation of Rolle's Theorem:
From ***Rolle's Theorem*** $\Rightarrow$ if a function is continuous on $[a,b]$ and differentiable on (a,b) and if $f(a)=f(b)$ then there must be at least one value of x between a and b at which the graph of has a horizontal tangent.