Question

Find the area of the region bounded by the graph of curves

$$y = x + 6$$
, $y = x^3$, $y = -\frac{x}{2}$

Solution

The graph of these curves looks like this one:

Blue line is y = x + 6, green is

 $y = -\frac{x}{2}$, orange curve is $y = x^3$. The point of intersection of blue and green line is determined by

$$x_1 + 6 = -\frac{x_1}{2}$$

$$x_1 = -4$$

The point of intersection of blue and orange line is determined by

 $x_2 + 6 = x_2^3 \quad \rightarrow \quad x_2 = 2$

Green and orange curves intersect at the point

$$x_3^3 = -\frac{x_3}{2} \rightarrow \quad x_3 = 0$$

Therefore, the area of bounded region is

$$S = \int_{-4}^{2} (x+6)dx - \int_{-4}^{0} \left(-\frac{x}{2}\right)dx - \int_{0}^{2} x^{3}dx$$

Evaluate it:

$$S = \frac{(2+6)^2}{2} - \frac{(-4+6)^2}{2} + \frac{0^2}{4} - \frac{(-4)^2}{4} - \frac{2^4}{4} + \frac{0^4}{4} = 22$$

Answer: *S* = 22.

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