Answer on Question #50970 - Math - Calculus

Find the area of the curve from the following equations:

$7x - 9y + 63 = 0$

$2x + y - 2 = 0$

$11x + 9y - 99 = 0$

Solution:

These lines bound the triangle. To find coordinates of its vertices we have to solve 3 linear systems:

$$\begin{cases} 2x + y - 2 = 0 \\ 11x + 9y - 99 = 0 \end{cases} \qquad \begin{cases} 7x - 9y + 63 = 0 \\ 2x + y - 2 = 0 \end{cases} \qquad \begin{cases} 7x - 9y + 63 = 0 \\ 11x + 9y - 99 = 0 \end{cases}$$

These linear systems have following solutions:

$$\begin{cases} x = -\frac{81}{7} \\ y = \frac{176}{7} \end{cases} \qquad \begin{cases} x = -\frac{9}{5} \\ y = \frac{28}{5} \end{cases} \qquad \begin{cases} x = 2 \\ y = \frac{77}{9} \end{cases}$$

The equations of given lines can be rewritten as follows:

$y = \frac{7}{9}x + 7$

$y = 2 - 2x$

$y = 11 - \frac{11}{9}x$

The area of triangle with vertices at these points is given by the following integral

$$S = \int_{-\frac{81}{7}}^{-\frac{9}{5}} \left(\left(11 - \frac{11}{9}x\right) - (2 - 2x)\right) \mathrm{d}x + \int_{-\frac{9}{5}}^{2} \left(\left(11 - \frac{11}{9}x\right) - \left(\frac{7}{9}x + 7\right)\right) \mathrm{d}x =$$

$$= \int_{-\frac{81}{7}}^{-\frac{9}{5}} \left(9 + \frac{7}{9}x\right) \mathrm{d}x + \int_{-\frac{9}{5}}^{2} (4 - 2x) \mathrm{d}x = \left(9x - \frac{7}{18}x^2\right)\Big|_{-\frac{81}{7}}^{-\frac{9}{5}} + (4x - x^2)\Big|_{-\frac{9}{5}}^{2} =$$

$$= -9\frac{9}{5} + \frac{7}{18}\left(\frac{9}{5}\right)^2 - \left(-9\frac{81}{7}\right) - \frac{7}{18}\left(\frac{81}{7}\right)^2 + 4 \cdot 2 - 2^2 + 4\frac{9}{5} + \left(\frac{9}{5}\right)^2 = \frac{361}{7}$$

Answer: $\frac{361}{7}$.