

Answer on Question #50939 - Math - Integral Calculus

Integrate with respect to t :

$$\int_0^4 t\sqrt{t-2} dt$$

Solution

The domain of the integrand is given by

$$t \in [2, +\infty],$$

since the square root of negative number doesn't exist (at least in real analysis). Therefore the given integral doesn't exist, since the integrand is integrated over the region $[0, 2)$ where it's not defined.

Answer: doesn't exist.

2) Using Newton-Leibnitz formula and integral formula for powers,

$$\begin{aligned} \int_2^4 t\sqrt{t-2} dt &= \int_2^4 (t-2)\sqrt{t-2} dt + 2 \int_2^4 \sqrt{t-2} dt = |u := t-2, du = dt| \\ &= \int_0^2 u\sqrt{u} du + 2 \int_0^2 \sqrt{u} du = \frac{u^{5/2}}{5/2} \Big|_0^2 + 2 \frac{u^{3/2}}{3/2} \Big|_0^2 = \frac{2}{5} \cdot 4\sqrt{2} + \frac{4}{3} \cdot 2\sqrt{2} = \frac{64\sqrt{2}}{15} \end{aligned}$$

3)

$$\begin{aligned} \int 0.4t\sqrt{t-2} dt &= 0.4 \int t\sqrt{t-2} dt = 0.4 \int (t-2)\sqrt{t-2} dt + 0.4 \cdot 2 \int \sqrt{t-2} dt = \\ |u := t-2, du = dt| &= 0.4 \int u\sqrt{u} du + 0.8 \int \sqrt{u} du = 0.4 \frac{u^{5/2}}{5/2} + 0.8 \frac{u^{3/2}}{3/2} + C, \text{ where } C \text{ is an} \\ &\text{arbitrary real constant.} \end{aligned}$$

4)

$$\begin{aligned} \int 0.4(t-2)\sqrt{t} dt &= 0.4 \int t\sqrt{t} dt - 0.4 \cdot 2 \int \sqrt{t} dt = 0.4 \int t\sqrt{t} dt - 0.8 \int \sqrt{t} dt = 0.4 \frac{t^{5/2}}{5/2} - \\ &0.8 \frac{t^{3/2}}{3/2} + C, \text{ where } C \text{ is an arbitrary real constant.} \end{aligned}$$

5)

$$\int_0^4 (t-2)\sqrt{t} dt = \int_0^4 t\sqrt{t} dt - 2 \int_0^4 \sqrt{t} dt = \frac{t^{5/2}}{5/2} \Big|_0^4 - 2 \frac{t^{3/2}}{3/2} \Big|_0^4 = \frac{2}{5} \cdot 32 - \frac{4}{3} \cdot 8 = \frac{32}{15}$$