Answer on Question #50934 – Math – Integral Calculus

Integrate with respect to x: ∫csc2xdx

Solution.

First we use Trigonometric Identity, namely Reciprocal Identity, namely: $\csc x = \frac{1}{\sin x}$.

According to Reciprocal Identity $\csc 2x = \frac{1}{\sin 2x}$

$$I = \int \csc 2x dx = \int \frac{1}{\sin 2x} dx \Rightarrow$$

Step 1. Let's multiply and divide the integrand, i.e. the function that is to be integrated, $\frac{1}{\sin 2x}$ by $\sin 2x$:

$$I = \int \csc 2x dx = \int \frac{1}{\sin 2x} dx = \int \frac{\sin 2x}{\sin^2 2x} dx = \left| \begin{matrix} \text{Pythagorean identity} \\ \sin^2 2x = 1 - \cos^2 2x \end{matrix} \right| = \int \frac{\sin 2x}{1 - \cos^2 2x} dx =$$

$$= -\frac{1}{2} \int \frac{d \cos 2x}{(1 - \cos^2 2x)} = |t = \cos 2x| = -\frac{1}{2} \int \frac{dt}{(1 - t^2)} = -\frac{1}{2} \int \frac{dt}{(1 - t^2)} \Rightarrow$$

Step 2. Let's use the method of Partial Fraction Decomposition.

1) Factor the denominator: $(1 - t^2) = (1 - t)(1 + t)$

2) Write one partial fraction for each of those factors:

3) $\frac{1}{(1 - t^2)} = \frac{A}{(1 - t)} + \frac{B}{(1 + t)} = \frac{A(1 + t) + B(1 - t)}{(1 - t^2)}$

4) Multiply through by the denominator so we no longer have fractions.
$A(1 + t) + B(1 - t) = 1$

5) Find unknown coefficients A and B. Substituting the roots ("zeros") of the denominator we will obtain:

$$\begin{cases} t = 1 & A(1+1) + B(1-1) = 1 \\ t = -1 & A(1+t) + B(1-t) = 1 \end{cases} \Rightarrow \begin{cases} 2A = 1 \\ 2B = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{2} \\ B = \frac{1}{2} \end{cases}$$

Step 3.

$$I = -\frac{1}{2} \int \frac{dt}{(1 - t^2)} = -\frac{1}{4} \int \frac{dt}{1 - t} - \frac{1}{4} \int \frac{dt}{1 + t} = +\frac{1}{4} \ln|1 - t| - \frac{1}{4} \ln|1 + t| + c = \frac{1}{4} \ln\left|\frac{1 - t}{1 + t}\right| + c \Rightarrow$$

Step 4. Now let's apply inverse substitution of variables

$I = \frac{1}{4} \ln\left|\frac{1 - \cos 2x}{1 + \cos 2x}\right| + c$, where c is an arbitrary real constant.

Answer: $I = \frac{1}{4} \ln\left|\frac{1 - \cos 2x}{1 + \cos 2x}\right| + c.$