

Answer on Question #50927 – Math – Integral Calculus

Find the $\int \tan^3 x \sec^3 x dx$

A $\tan 2x + 1$

B $\cot 2x + 1$

C $\sec 2x + 1$

D $\sec 2x$

Solution

$$\begin{aligned} I &= \int \tan^3 x \cdot \sec^3 x dx = \\ &= \int \tan x \cdot \tan^2 x \cdot \sec^3 x dx = \int (\tan x \cdot \sec^2 x - \tan x) \sec^3 x dx \\ &= \\ &= \int (\sec^5 x \cdot \tan x - \sec^3 x \cdot \tan x) dx = \int (\sec^4 x - \sec^2 x) \sec x \cdot \tan x dx = \\ &= \left\{ u = \sec x = \frac{1}{\cos x}, \quad du = -\frac{-\sin x dx}{\cos^2 x} = \tan x \cdot \sec x dx \right\} = \\ &= \int (u^4 - u^2) du = \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C, \end{aligned}$$

where C is an arbitrary real constant.