Answer on Question #50901 – Math – Differential Calculus | Equations

a) Solve the following equation:

$(D^2 - 2DD' + D'^2)\, z = tan\,(y + x)$

b) Solve: $z\,(p - q) = z^2 + (x + y)^2$

Solution

a) $(D^2 - 2DD' + D'^2)\, z = tan\,(y + x)$ or $(D - D')^2\, z = tan\,(y + x)$

Here auxiliary equation is $(m - 1)^2 = 0$ so that $m = 1$.

Thus

$C.F. = \varphi_1(y + x) + x\varphi_2(y + x)$, where $\varphi_1\ and\ \varphi_2$ are arbitrary functions.

Now

$$P.I. = \frac{1}{(D - D')^2}\tan(y + x) = \frac{x^2}{1^2 2!}\tan(y + x) = \frac{x^2}{2}\tan(y + x).$$

The general solution is

$$z = \varphi_1(y + x) + x\varphi_2(y + x) + \frac{x^2}{2}\tan(y + x).$$

b) The auxiliary equations are

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x + y)}.$$

Taking the first two members we get

$$-zdx = zdy \to x + y = C_1.$$

Thus

$$dx(z^2 + C_1) = zdz \to 2dx = \frac{d(z^2 + C_1)}{(z^2 + C_1)} = d(\ln(z^2 + C_1)) \to x - \ln(\sqrt{z^2 + C_1}) = C_2.$$

$$z = \varphi_1(y + x) + \varphi_2\left(x - \ln(\sqrt{z^2 + x + y})\right).$$

Where $\varphi_1\ and\ \varphi_2$ are arbitrary functions.