Answer on Question #50897-Math-Differential Calculus-Equations

a) A wet porus substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour, then find the time when it has lost 95% moisture provided the weather conditions remain the same.

b) If the interest is compounded continuously, the amount of money invested increases at a rate proportional to its size. Let Rs. 10,000 be invested at 10% compounded continuously. Then in how many years will the original investment double itself?

c) Solve: dy/dx + (x/1-x^2) y = xy, y(0)=1.

Solution

a)

$$\frac{dM}{dt} = -kM \rightarrow M = M_0 e^{-kt}.$$

$$M(1\ hour) = \frac{1}{2}M_0 \rightarrow \frac{1}{2} = e^{-k} \rightarrow k = \ln 2\ hours^{-1}.$$

$$M(T) = M_0(1-0.95) = 0.05M_0 = M_0 e^{-kT}.$$

$$T = \frac{1}{k}\ln 20 = \frac{\ln 20}{\ln 2} = 4.3\ hours.$$

Answer: $\mathbf{4.3\ hours}$.

b) $r = 0.1.$

$$R = R_0 e^{rt}.$$

$$R(T) = 2R_0 = R_0 e^{rT}.$$

$$T = \frac{1}{r}\ln 2 = \frac{\ln 2}{0.1} = 6.9\ years.$$

Answer: 6.9 years.

c)

$$\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right) y \ =\ x\sqrt{y}, y(0) = 1.$$

$$t^2 = y \rightarrow 2t\frac{dt}{dx} + \left(\frac{x}{1-x^2}\right)t^2 = xt.$$

$$2\frac{dt}{dx} + \left(\frac{x}{1-x^2}\right)t = x.$$

General solution:

$$2\frac{dt}{dx} + \left(\frac{x}{1-x^2}\right)t = 0 \rightarrow \frac{2dt}{t} = -\left(\frac{xdx}{1-x^2}\right) = \frac{1}{2}\frac{d(1-x^2)}{1-x^2} \rightarrow \ln\frac{t^2}{t_0^2} = \frac{1}{2}\ln(1-x^2).$$

$t^2 = t_o^2\sqrt{1-x^2} \to t = t_0\sqrt[4]{1-x^2}.$

Partial solution is

$t = -\frac{1}{3}(1-x^2).$

Thus

$y = \left(t_0\sqrt[4]{1-x^2} - \frac{1}{3}(1-x^2)\right)^2, y(0) = 1.$

$1 = \left(t_0 - \frac{1}{3}\right)^2 \to t_0 = \frac{4}{3}.$

Answer: $\left(\frac{4}{3}\sqrt[4]{1-x^2} - \frac{1}{3}(1-x^2)\right)^2.$