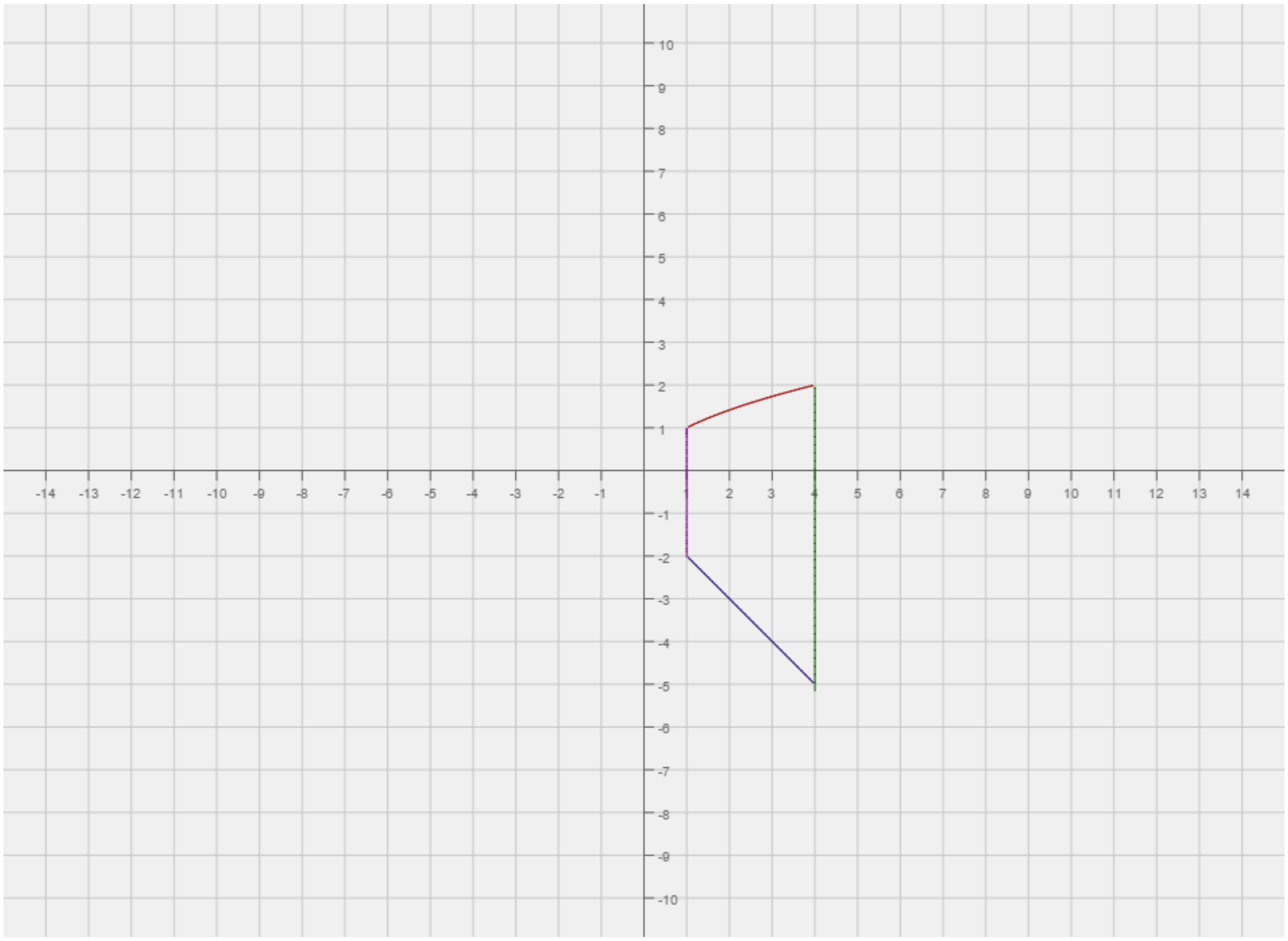
Answer on Question #50829 – Math – Integral Calculus

Find the area of the region bounded by the graphs of $y=\sqrt{x}$ and $y=-x-1$ between x=1 and x=4.

Solution



First method

To find the area of region, we have to evaluate the following double integral:

$$\iint_A dxdy=\int_1^4 dx\int_{-x-1}^{\sqrt{x}} dy=\int_1^4(\sqrt{x}+x+1)dx=\int_1^4\sqrt{x}dx+\int_1^4 xdx+\int_1^4 dx=$$

$$=\frac{2}{3}x^{3/2}\Big|_1^4+\frac{1}{2}x^2\Big|_1^4+x\Big|_1^4=\frac{16}{3}-\frac{2}{3}+8-\frac{1}{2}+3=\frac{91}{6}=15.1667$$

where A is a region bounded by $y=\sqrt{x}$ and $y=-x-1$ between x=1 and x=4.

Second method

Let S be the area of the given region.

We have to find the sum $\int_1^4 \sqrt{x}dx + \left|\int_1^4 (-x-1)dx\right|$. The absolute value of the definite integral in the second term is taken, because all corresponding values of integrand are negative.

$$\int_1^4 \sqrt{x}dx = \frac{2}{3}x^{3/2}\Big|_1^4 = \frac{14}{3}$$

$$\int_1^4 (-x-1)\,dx = -\frac{x^2}{2}\Big|_1^4 - x\Big|_1^4 = -8 + \frac{1}{2} - 3 = -\frac{21}{2}$$

$$S = \frac{14}{3} + \left|-\frac{21}{2}\right| = \frac{14}{3} + \frac{21}{2} = \frac{91}{6} = 15.1667$$

Answer: $\frac{91}{6}$ *or* 15.1667.