Answer on Question #50806 – Math – Differential Calculus | Equations

Question

y=x+(1/x). Can we find any other maxima or minima at any other point except x=1 and -1? I sketch the graph , at x=1 there is a minima and x=-1 there is a maxima, but cannot understand from graph why the maxima is less than the minima.

Solution

If $y'(x_n) = 0$ and $y''(x_n) < 0$, then y(x) has a relative maximum at x_n.

If $y'(x_n) = 0$ and $y''(x_n) > 0$, then y(x) has a relative minimum at x_n.

See http://en.wikipedia.org/wiki/Second_derivative_test.

Function $y = x + \frac{1}{x}$ is not defined at $x = 0$.

$$y = x + \frac{1}{x}$$

$$y' = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}; y' = 0 \Rightarrow \left[\begin{array}{l} x_1 = 1 \\ x_2 = -1 \end{array}\right.$$

$$y'' = \frac{2}{x^3}$$

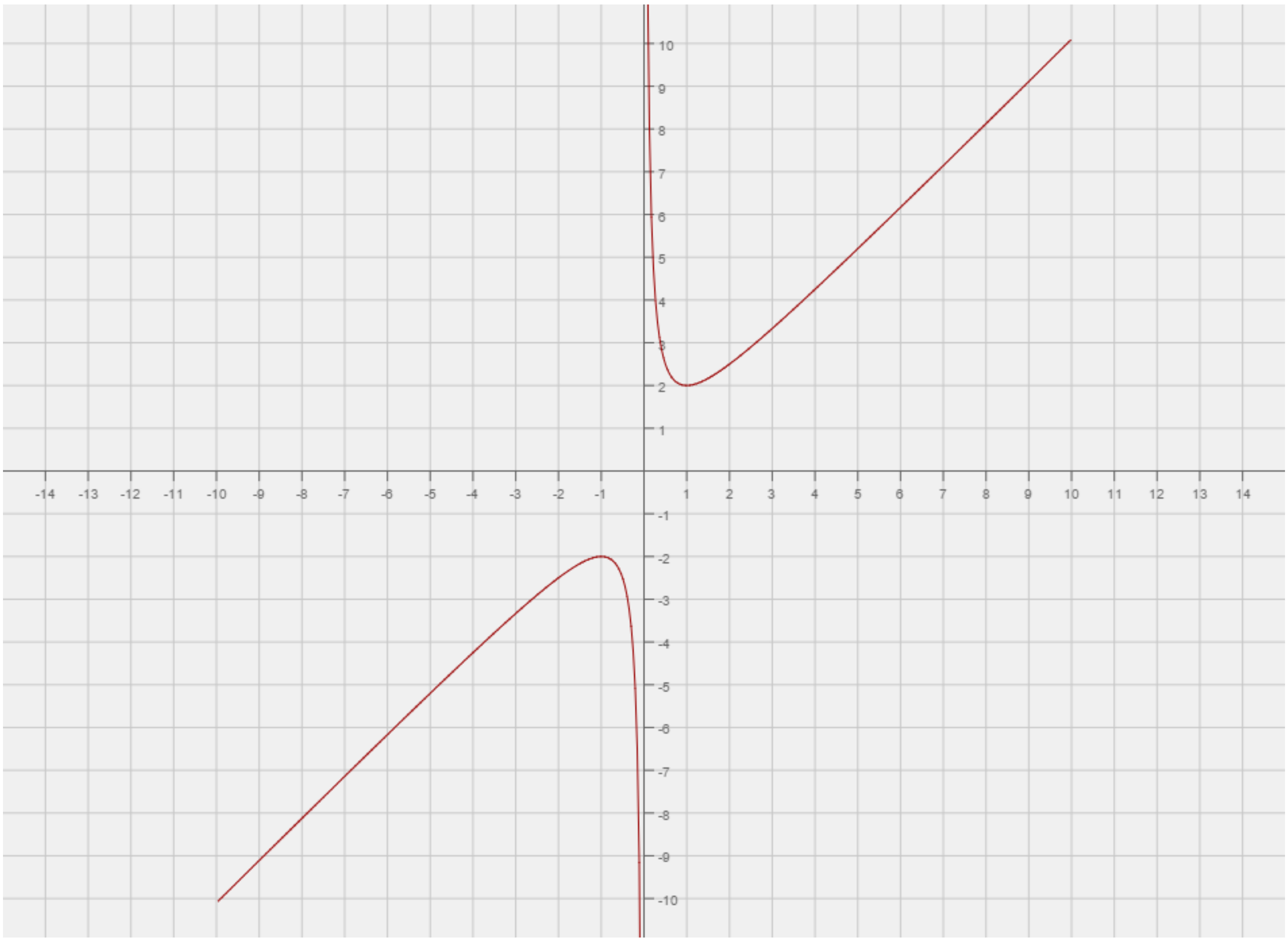
Hence, the critical values are $x_1 = 1$ and $x_2 = -1$.

Now let us compute $y''(x_n)$:

$y''(x_1) = y''(1) = \frac{2}{1^3} = 2 > 0$, so y(x) has a relative minimum at $x_1=1$

$y''(x_2) = y''(-1) = \frac{2}{(-1)^3} = -2 < 0$, so y(x) has a relative maximum at $x_1= -1$.

Global minimum and maximum are $-\infty$ and $+\infty$ respectively.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