Answer on Question #50801 – Math – Calculus

Calculate the area of the bounded region between the curves $y^2 = x$ and $3y = -3y + 9$.

Solution

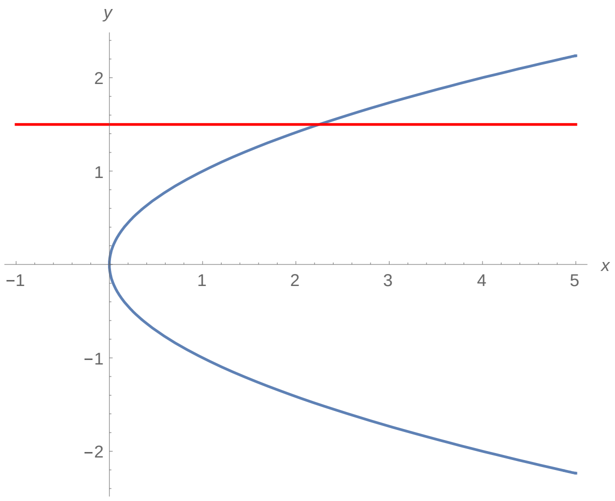
First of all, we note that most likely, there is a mistake in the statement. Really, we can rewrite:

$$3y = -3y + 9$$

$$6y = 9$$

$$y = \frac{3}{2}$$

In this case, we have such illustration:



As we can see, there is no bounded region between the curves.

So, we will solve this problem:

Calculate the area of the bounded region between the curves $y^2 = x$ and $3y = -3x + 9$.

$$3y = -3x + 9$$

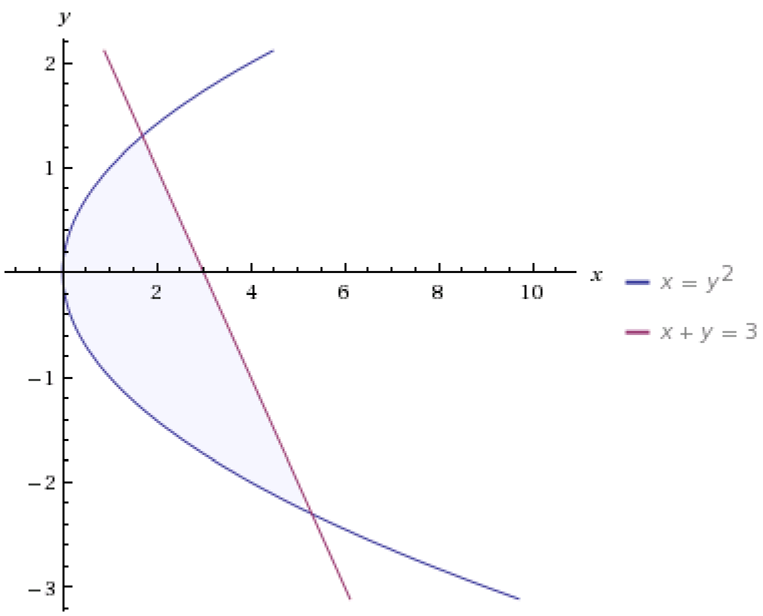
$$y = -x + 3$$

Then find intersection points of these curves:

$$\begin{cases} y^2 = x \\ y = -x + 3 \end{cases}$$

$$(-x+3)^2 = x$$

$$x^2-6x+9=x$$
$$x^2-7x+9=0$$
$$D=49-36=13$$
$$x_1=\frac{7-\sqrt{13}}{2}\Rightarrow y_1=3-\frac{7-\sqrt{13}}{2}=\frac{\sqrt{13}-1}{2};$$
$$x_2=\frac{7+\sqrt{13}}{2}\Rightarrow y_2=3-\frac{7+\sqrt{13}}{2}=\frac{-\sqrt{13}-1}{2};$$



The area of the bounded region between the curves can be found as the following integral:

$$\int_{y_1}^{y_2}\left(-y+3-y^2\right)dy=\int_{\frac{-\sqrt{13}-1}{2}}^{\frac{\sqrt{13}-1}{2}}\left(-y+3-y^2\right)dy=\left(-\frac{y^3}{3}-\frac{y^2}{2}+3y\right)\Bigg|_{\frac{-\sqrt{13}-1}{2}}^{\frac{\sqrt{13}-1}{2}}=$$

$$=-\frac{\left(\frac{\sqrt{13}-1}{2}\right)^3}{3}-\frac{\left(\frac{\sqrt{13}-1}{2}\right)^2}{2}+3\frac{\sqrt{13}-1}{2}+\frac{\left(\frac{-\sqrt{13}-1}{2}\right)^3}{3}+\frac{\left(\frac{-\sqrt{13}-1}{2}\right)^2}{2}-3\left(\frac{-\sqrt{13}-1}{2}\right)=$$

$$=\frac{13\sqrt{13}}{6}\approx 7{,}812$$

Answer: $\frac{13\sqrt{13}}{6}\approx 7{,}812$.