## Answer on Question #50739 – Math – Differential Calculus | Equations

Solve the initial value problem: d^2x /dt^2 + 2 dy/dx - 3x = 0, X(2pie) = 1, x^.(2pie) = 13

### Solution

$$x'' + 2x' - 3x = 0, \qquad x(2\pi) = 1, \qquad x'(2\pi) = 13.$$

**Characteristic equation is** $r^2 + 2r - 3 = 0$, **hence its solutions are**

$$r = -3 \ \ or \ \ r = 1.$$

**So,** $x(t) = c_1 e^{-3t} + c_2 e^t$**, hence** $x'(t) = -3c_1 e^{-3t} + c_2 e^t$**.**

**From conditions**

$$x(2\pi) = 1, \qquad x'(2\pi) = 13$$

**obtain the following system of linear equations with respect to** $c_1$ **and** $c_2$**,**

**which to be solved by Cramer's method :**

$$\rightarrow \begin{cases} c_1 e^{-6\pi} + c_2 e^{2\pi} = 1 \\ -3c_1 e^{-6\pi} + c_2 e^{2\pi} = 13 \end{cases} \rightarrow \begin{cases} c_1 = \dfrac{\begin{vmatrix} 1 & e^{2\pi} \\ 13 & e^{2\pi} \end{vmatrix}}{\begin{vmatrix} e^{-6\pi} & e^{2\pi} \\ -3e^{-6\pi} & e^{2\pi} \end{vmatrix}} \\ c_2 = \dfrac{\begin{vmatrix} e^{-6\pi} & 1 \\ -3e^{-6\pi} & 13 \end{vmatrix}}{\begin{vmatrix} e^{-6\pi} & e^{2\pi} \\ -3e^{-6\pi} & e^{2\pi} \end{vmatrix}} \end{cases}$$

$$\rightarrow \begin{cases} c_1 = \dfrac{-12e^{2\pi}}{4e^{-4\pi}} \\ c_2 = \dfrac{16e^{-6\pi}}{4e^{-4\pi}} \end{cases} \rightarrow \begin{cases} c_1 = -3e^{6\pi} \\ c_2 = 4e^{-2\pi} \end{cases}$$

**Thus,** $x(t) = -3e^{6\pi - 3t} + 4e^{t - 2\pi}$ **is the solution to the initial value problem.**