Answer on Question#50737

1) Obtain all the first and second order partial derivatives of the function: $f(x,y)=\ln(1+xy^2)$

Solution. Let's compute first order partial derivatives of the origin function:

$$\frac{\partial}{\partial x}f(x,y)=\frac{y^2}{1+xy^2}; \qquad \frac{\partial}{\partial y}f(x,y)=\frac{2xy}{1+xy^2}$$

Let's compute second order partial derivatives of the origin function:

$$\frac{\partial^2}{\partial x^2}f(x,y)=\frac{\partial}{\partial x}\left(\frac{y^2}{1+xy^2}\right)=\frac{\partial}{\partial x}\left(y^2(1+xy^2)^{-1}\right)=-y^2(1+xy^2)^{-2}y^2=\frac{-y^4}{\left(1+xy^2\right)^2};$$

$$\frac{\partial^2}{\partial x\partial y}f(x,y)=\frac{\partial^2}{\partial y\partial x}f(x,y)=\frac{\partial}{\partial y}\left(\frac{y^2}{1+xy^2}\right)=\frac{2y\left(1+xy^2\right)-2xy^3}{\left(1+xy^2\right)^2}=\frac{2y}{\left(1+xy^2\right)^2};$$

$$\frac{\partial^2}{\partial y^2}f(x,y)=\frac{\partial}{\partial y}\left(\frac{2xy}{1+xy^2}\right)=\frac{2x\left(1+xy^2\right)-4x^2y^2}{\left(1+xy^2\right)^2}=\frac{2x-2x^2y^2}{\left(1+xy^2\right)^2}.$$