Answer on Question #50731 - Math - Algebra

The greater of the two angles A = 2tan^-1((2√2)-1) and B=3sin^-1(1/3) + sin^-1(3/5) is?

Solution:

Remember that $\sqrt{2} \approx 1.4$. Because $\left(2\sqrt{2}-1\right)^2 = 8 - 4\sqrt{2} + 1 = 9 - 4\sqrt{2} > 3 \Leftrightarrow$ $6 > 4\sqrt{2} \Leftrightarrow \text{raising to the second power} \Leftrightarrow 36 > 32$, which is true, it follows that $2\sqrt{2} - 1 > \sqrt{3}$.

According to this fact and $tan\,\pi/3 = \sqrt{3}$, obtain that $\left(2\sqrt{2}-1\right) > tan\,\pi/3$, then $2tan^{-1}\left(2\sqrt{2}-1\right) > 2\,\pi/3$, because the inverse tangent function is strictly increasing function. Thus, $A > 2\,\pi/3$.

Remember that $sin3\alpha = 3sin\alpha - 4sin^3\alpha$, then $3\alpha = sin^{-1}[3sin\alpha - 4sin^3\alpha]$.

We have $\alpha = sin^{-1}\frac{1}{3}$ and $sin\alpha = \frac{1}{3}$

Thus,

$$B = 3sin^{-1}\frac{1}{3} + sin^{-1}\frac{3}{5} = sin^{-1}\left[3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3\right] + sin^{-1}\frac{3}{5} = sin^{-1}\frac{23}{27} + sin^{-1}\frac{3}{5}$$

Note that $\frac{23}{27} < \frac{\sqrt{3}}{2}$ and $\frac{3}{5} < \frac{\sqrt{3}}{2}$, because $(23\cdot 2)^2 < \left(27\sqrt{3}\right)^2$ and $(3\cdot 2)^2 < \left(5\sqrt{3}\right)^2$, which is equivalent to $2116 < 2187$ and $36 < 75$, true inequalities.

So $sin^{-1}\frac{23}{27} + sin^{-1}\frac{3}{5} < sin^{-1}\frac{\sqrt{3}}{2} + sin^{-1}\frac{\sqrt{3}}{2} = \pi/3 + \pi/3 = 2\,\pi/3$, because the inverse sine function is strictly increasing.

Thus, $A > 2\,\pi/3$, $B < 2\,\pi/3$, so A > B.

Answer: A > B.