Answer on Question #50714 – Math – Differential Calculus | Equations

How to find maxima and minima. Process

1. to find x where dy/dx=o. let the x =a

2. then find d2y/dx2

3.if d2y/dx2 >0 at x=a then we will find a minimum for x=a

4.if d2y/dx2 <0 at x=a then we will find a maximum for x=a

5.if d2y/dx2=0 at x=a then we will have to proceed to the d3y/dx3

6.if d3y/dx3$\neq$ 0 at x=a, we will not find any maxima or minima.

7.if d3y/dx3=0 at x=a then we will proceed to d4y/dx4 and if d4y/dx4 >0 at x=a then it's minimum and if d4y/dx4 < 0 at x=a then it is maximum.

Please explain the rule 6 above. Why will we not find any maxima or minima in that case? Please explain with figure and example.

Solution.

6. Let P be the Taylor polynomial of y(x) around $x = a$ of order 3.

Then, $P(x) = k(x-a)^3$ $(because\ \ y'(a) = y''(a) = 0)$

and so $y(x) = k(x-a)^3 + O(x-a)^4$.

It follows that sufficiently close to $x = a$ $y(x)$ has the same sign as $k(x-a)^3$

and behaves like $k(x-a)^3$ and strictly increasing $(k > 0)$ or decreasing

$(k < 0)$ and therefore has not maxima or minima at $x = a$.

7. Let P be the Taylor polynomial of y(x) around $x = a$ of order 4.

Then, $P(x) = k(x-a)^4$ $(because\ \ y'(a) = y''(a) = y'''(a) = 0)$

and so $y(x) = k(x-a)^4 + O(x-a)^5$

It follows that sufficiently close to $x = a$ $y(x)$ behaves like $k(x-a)^4$ and has maxima $(k < 0)$ or minima $(k > 0)$.