**Answer on Question #50713 – Math – Calculus**

please provide me some problem, where third derivative or more than that is used to find maxima and minima. and please solve one problem like this.

**Solution**

**Let** $y(x) = x^4 + x^5,\ y' = 4x^3 + 5x^4,\ y'' = 12x^2 + 20x^3,$

$y''' = 24x + 60x^2,\ y^{IV} = 24 + 120x.$

**Solve** $y' = 0$**, that is,** $4x^3 + 5x^4 = 0$**, hence** $x^3(4 + 5x) = 0$ **and finally** $x = 0$ **or** $x = -\frac{4}{5}$ **are critical points. Thus,** $y'(0) = 0$ **and** $y'\left(-\frac{4}{5}\right) = 0.$

**Evaluate** $y''\left(-\frac{4}{5}\right) = 12\left(-\frac{4}{5}\right)^2 + 20\left(-\frac{4}{5}\right)^3 = \frac{12\cdot 16 - 20\cdot 64/5}{25} = \frac{12\cdot 16 - 4\cdot 64}{25} < 0,$ **hence** $x = -\frac{4}{5}$ **is a local maximum.**

**Next,** $y''(0) = 0.$ **To classify** $x = 0$**, further research is required.**

**So,** $y(0) = y'(0) = y''(0) = y'''(0) = 0,\ y^{IV}(0) > 0.$

**As we can see,** $n = 4$ **is even and** $y^{IV}(0) > 0$**. Sufficiently close to** $x = 0, y(x)$ **strictly decreases when** $x \to 0$ **from the left and from the right and therefore function** $y(x) = x^4 + x^5$ **has local minimum at** $x = 0.$

**In case of odd order *n*, point cannot be a point of extremum.**