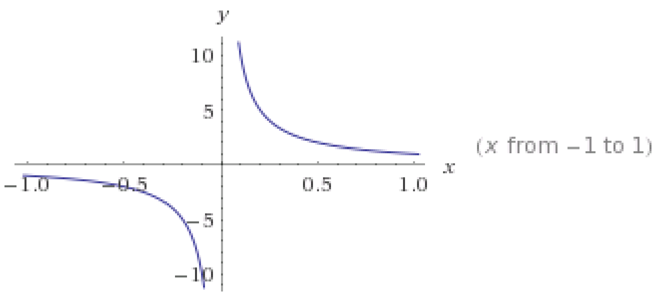
Answer on Question #50712 – Math – Calculus

What is the graph of y=x+(1/x). I can't sketch the graph, because if i use x=0 then y=infinity.

and i want to find maxima and minima of the function. Here the maxima is less than in minima, but don't understand why this happens? Please show me the maxima and minima in graph.

Solution

Function $y = x + \frac{1}{x}$ is not defined at point $x = 0$. Near $x = 0$ function acts just like hyperbole.



With singularity in the point $x = 0$

$y = x + \frac{1}{x} \sim \frac{1}{x}$, as $x \to 0$, because

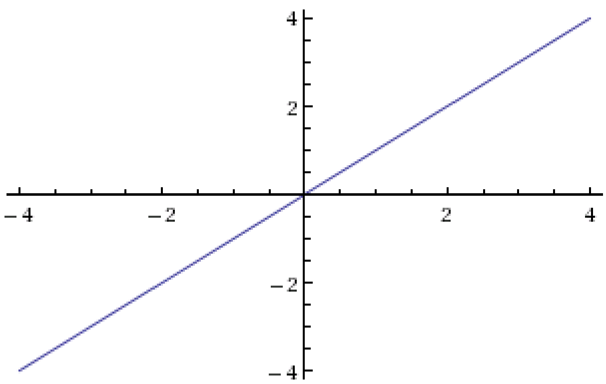
$$\lim_{x\to 0} \frac{x+\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to 0}\left(\frac{x^2+1}{x} \cdot x\right) = \lim_{x\to 0}(x^2 + 1) = 1.$$

Straight line $x = 0$ is vertical asymptote of the graph of the function $y = x + \frac{1}{x}$, because

$$\lim_{x\to 0}\left(x + \frac{1}{x}\right) = \lim_{x\to 0}(x) + \lim_{x\to 0}\left(\frac{1}{x}\right) = 0 + \lim_{x\to 0}\left(\frac{1}{x}\right) = \infty.$$

See https://en.wikipedia.org/wiki/Asymptote .

With larger values of x function acts like linear function.



With singularity in the points $x = \pm\infty$

$y = x + \frac{1}{x} \sim x$, as $x \to +\infty$,

$y = x + \frac{1}{x} \sim x$, as $x \to -\infty$,

because $\lim\limits_{x\to+\infty} \frac{x+\frac{1}{x}}{x} = \lim\limits_{x\to+\infty} \left(1 + \frac{1}{x^2}\right) = \lim\limits_{x\to+\infty} 1 + \lim\limits_{x\to+\infty} \frac{1}{x^2} = 1 + 0 = 1,$

$\lim\limits_{x\to-\infty} \frac{x+\frac{1}{x}}{x} = \lim\limits_{x\to-\infty} \left(1 + \frac{1}{x^2}\right) = \lim\limits_{x\to-\infty} 1 + \lim\limits_{x\to-\infty} \frac{1}{x^2} = 1 + 0 = 1.$

Straight line $y = x$ is oblique asymptote of the graph of the function $y = x + \frac{1}{x}$, because

$$k = \lim_{x\to+\infty} \frac{\left(x + \frac{1}{x}\right)}{x} = \lim_{x\to+\infty} (1) + \lim_{x\to+\infty} \left(\frac{1}{x^2}\right) = 1 + 0 = 1,$$

$$b = \lim_{x\to+\infty} (f(x) - kx) = \lim_{x\to+\infty} \left(x + \frac{1}{x} - kx\right) = \lim_{x\to+\infty} \left(x + \frac{1}{x} - x\right) = \lim_{x\to+\infty} \left(\frac{1}{x}\right) = 0$$

and

$k = \lim\limits_{x\to-\infty} \frac{\left(x+\frac{1}{x}\right)}{x} = \lim\limits_{x\to-\infty} (1) + \lim\limits_{x\to-\infty} \left(\frac{1}{x^2}\right) = 1 + 0 = 1,$

$b = \lim\limits_{x\to-\infty} \left(x + \frac{1}{x} - kx\right) = \lim\limits_{x\to-\infty} \left(x + \frac{1}{x} - x\right) = \lim\limits_{x\to-\infty} \left(\frac{1}{x}\right) = 0,$

where k is the slope and b is the intercept of asymptote $y = kx + b$.

See https://en.wikipedia.org/wiki/Asymptote .

To find local maxima and minima, solve

$y' = 0 \Longrightarrow \left(x + \frac{1}{x}\right)' = 0 \Rightarrow 1 - \frac{1}{x^2} = 0 \Longrightarrow \frac{x^2-1}{x^2} = 0 \Longrightarrow \frac{(x-1)(x+1)}{x^2} = 0 \Longrightarrow$

$\Longrightarrow x = -1$ or $x = 1$.

Local minimum here is the point where $x = 1$, because $y'(x) < 0$ when $-1 < x < 1$ and $y'(x) > 0$ when $x > 1$.

Local maximum here is the point where $x = -1$, because $y'(x) > 0$ when $x < -1$ and $y'(x) < 0$ when $-1 < x < 1$.

See http://en.wikipedia.org/wiki/First_derivative_test .

Though the real maximum is the infinity and the real minimum is the negative infinity.

Point $x = 1$ is local minimum, not global, because, for example,

$$y(-2) = -2 - \frac{1}{2} < y(1) = 1 + \frac{1}{1} = 2\,.$$

Here is the sketch of graph of function $= x + \frac{1}{x}$: