**Answer on Question #50710 – Math – Differential Calculus | Equations**

What is the differentiation of

$$f(x) = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)\ ?$$

**Solution**

Because (see http://en.wikipedia.org/wiki/Inverse_trigonometric_functions)

$$(\sec^{-1} t)' = \frac{1}{|t|\sqrt{t^2-1}}, |t| > 1$$

Then using the chain and quotient rules for differentiation, find

$$f'(x) = \left(\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)\right)' = \frac{1}{\left|\frac{x^2+1}{x^2-1}\right|\sqrt{\left(\frac{x^2+1}{x^2-1}\right)^2-1}} \cdot \left(\frac{x^2+1}{x^2-1}\right)' =$$

$$= \frac{|x^2-1|}{|x^2+1|} \cdot \frac{|x^2-1|}{\sqrt{(x^2+1)^2-(x^2-1)^2}} \cdot \frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2} =$$

$$= \frac{(x^2-1)^2}{(x^2+1)\sqrt{(x^2+1)^2-(x^2-1)^2}} \cdot \frac{-4x}{(x^2-1)^2} =$$

$$= -\frac{2x}{(x^2+1)|x|} = \begin{cases} -\dfrac{2}{x^2+1}, when\ x > 0 \\ \dfrac{2}{x^2+1}, when\ x < 0 \end{cases}$$

In fact, $\left|\frac{x^2+1}{x^2-1}\right| > 1$ for all real $x, x \neq 0$.

**Answer:**

$$\left(\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)\right)' = -\frac{2x}{(x^2+1)|x|}\ .$$