Answer to Question 50683, Math, Differential Calculus | Equations

Why do we use $x = 0$ to find $f(0)$ in Maclaurin series? Why another numbers are not used? And why $f(0)$ is needed?

The answer to this question comes from the Taylor's theorem. If the function is infinitely differentiated in the neighborhood of the point $x = x_0$, then it can be represented as

$$f(x) = f(x_0) + f'(x_0)\frac{(x - x_0)}{1!} + f''(x_0)\frac{(x - x_0)^2}{2!} + .. + f^{(k)}(x_0)\frac{(x - x_0)^k}{k!} + ..$$

This theorem is a generalization of Lagrange's theorem

$$f(x) - f(x_0) = f'(\xi)(x - x_0)$$

Thus, Maclaurin series ($x_0 = 0$) is only a particular case of Taylor series. Saying rigorously, if we use $x = x_0$, we must call it Taylor series (not Maclaurin series).

Actually, we can use any point $x = x_0$ to write an expansion, but only when this function at the point satisfies the Taylor's theorem.