# Answer on Question #50682 – Math – Calculus

integral sqrt(1-x^2) . sin^-1 x dx

## Solution

## Method 1

**(integration by parts)**

We have $\sin^{-1} x = \arcsin x$ and $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$.

We use also integration by parts

$$\int \varphi(x) * \psi'(x)dx = \varphi(x) * \psi(x) - \int \psi(x) * \varphi'(x)dx.$$

Let $I = \int \sqrt{1-x^2} * \sin^{-1} x\, dx$.

Then,

$$I = \int \sqrt{1-x^2} \cdot \sin^{-1} x\, dx =$$

$$= \left| \varphi(x) = \sqrt{1-x^2} \cdot \sin^{-1} x, \psi'(x) = 1, \right.$$

$$\left. \varphi'(x) = \frac{-2x \sin^{-1} x}{2\sqrt{1-x^2}} + \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} = 1 - \frac{x \sin^{-1} x}{\sqrt{1-x^2}}, \psi(x) = x \right| =$$

$$= x\left(\sin^{-1} x \cdot \sqrt{1-x^2}\right)$$

$$- \int x\left(\sqrt{1-x^2} * \sin^{-1} x\right)' dx =$$

$$= x \sin^{-1} x \cdot \sqrt{1-x^2}$$

$$- \int x\left(\frac{-x \sin^{-1} x}{\sqrt{1-x^2}} + \sqrt{1-x^2} * \frac{1}{\sqrt{1-x^2}}\right) dx =$$

$$= x \sin^{-1} x \cdot \sqrt{1-x^2} - \int x dx - \int \frac{1-x^2-1}{\sqrt{1-x^2}} \sin^{-1} x\, dx = x \sin^{-1} x$$

$$\cdot \sqrt{1-x^2} - \frac{x^2}{2} - \int \sqrt{1-x^2} \cdot \sin^{-1} x\, dx + \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = x \sin^{-1} x$$

$$\cdot \sqrt{1-x^2} - \frac{x^2}{2} - I + \int \sin^{-1} x\, d(\sin^{-1} x) = x \sin^{-1} x \cdot \sqrt{1-x^2} - \frac{x^2}{2} - I$$

$$+ \frac{(\sin^{-1} x)^2}{2} + C,$$

where $C$ is an arbitrary real constant. Equating the left-hand and the right-hand sides of the previous equalities,

$$I = x \sin^{-1} x \cdot \sqrt{1-x^2} - \frac{x^2}{2} - I + \frac{(\sin^{-1} x)^2}{2} + C,$$

we obtain

$$2I = 2\int \sqrt{1-x^2} * \sin^{-1} x \, dx = x \sin^{-1} x \cdot \sqrt{1-x^2} - \frac{x^2}{2} + \frac{(\sin^{-1} x)^2}{2} + C$$

Hence

$$\int \sqrt{1-x^2} \cdot \sin^{-1} x \, dx = \frac{x \sin^{-1} x}{2} \sqrt{1-x^2} - \frac{x^2}{4} + \frac{(\sin^{-1} x)^2}{4} + C.$$

## Method 2

**(substitution and integration by parts)**

$$\int \sqrt{1-x^2} * \sin^{-1} x \, dx = |\text{substitution } \mathrm{x} = \sin(\mathrm{t}), \mathrm{t} = sin^{-1}(\mathrm{x}), \mathrm{dx} = \mathrm{costdt}\ | =$$

$$= \int \cos(t)\, sin^{-1}(\sin(t)) \cos(t)\, dt = \int tcos^2 tdt = \int t \frac{1+\cos(2t)}{2} dt$$

$$= \frac{1}{2}\int tdt + \frac{1}{2}\int tcos(2t)dt = \frac{1}{2}\frac{t^2}{2}$$

$$+ \frac{1}{4}\int tdsin(2t) = |integration\ by\ parts| = \frac{t^2}{4} + \frac{1}{4}\left(tsin(2t) - \int \sin(2t)\, dt\right)$$

$$= \frac{t^2}{4} + \frac{tsin(2t)}{4} + \frac{\cos(2t)}{8} + C = \frac{t^2}{4} + \frac{2tsin(t)\cos(t)}{4} + \frac{1-2sin^2 t}{8} + C =$$

$$= \frac{(sin^{-1}x)^2}{4} + \frac{sin^{-1}x \cdot sin(sin^{-1}x) \cdot cos(sin^{-1}x)}{2} + \frac{1-2\left(sin(sin^{-1}x)\right)^2}{8} + C =$$

$= \frac{\left(sin^{-1}x\right)^2}{4} + \frac{xsin^{-1}x \cdot \sqrt{1-x^2}}{2} - \frac{x^2}{4} + C_1$, where $C_1 = \frac{1}{8} + C$, later we can note $C$ instead of $C_1$ in the answer.

## Answer:

$$\boldsymbol{\int \sqrt{1-x^2} \cdot \sin^{-1} x \, dx = \frac{x \sin^{-1} x}{2} \sqrt{1-x^2} - \frac{x^2}{4} + \frac{(\sin^{-1} x)^2}{4} + C.}$$