Answer on Question #50681 – Math – Calculus

integral sec^-1 x dx

Solution

$$I = \int sec^{-1}(x)dx = \int udv = uv - \int v\,du$$

$where\ u = sec^{-1}(x),\ dv = dx,\ du = \frac{dx}{x\sqrt{x^2-1}},\ v = x.$

So, $I = xsec^{-1}(x) - \int \frac{dx}{\sqrt{x^2-1}}$

$$I_1 = \int \frac{dx}{\sqrt{x^2-1}}$$

$substitution\ x = sec(t), \qquad dx = sec(t)tan(t)dt$

$$I_1 = \int sec(t)dt = \int \frac{sec^2(t) + sec(t)tan(t)}{sec(t) + tan(t)}dt$$

$substitution\ s = sec(t) + tan(t), \qquad ds = [sec^2(t) + sec(t)tan(t)]dt$

$I_1 = \int \frac{ds}{s} = ln|s| + C = ln|sec(t) + tan(t)| + C = ln\left(x + \sqrt{x^2-1}\right) + C,$

where C **is an arbitrary real constant.**

Thus $I = xsec^{-1}(x) - I_1 = xsec^{-1}(x) - ln\left(x + \sqrt{x^2-1}\right) + C.$