Answer on Question #50670 – Math - Differential Calculus | Equations

Find the value of b for which the equation

(y e^2xy + x) dx + bx e^2xy dy = 0

is exact, and hence solve it for that value of b .

Solution

$$(ye^{2xy} + x) + bxe^{2xy}\frac{dy}{dx} = 0$$

If the differential equation is exact, then

$$\frac{\partial}{\partial x}(bxe^{2xy}) = \frac{\partial}{\partial y}(ye^{2xy} + x) \rightarrow \quad be^{2xy} + 2bxye^{2xy} = e^{2xy} + 2xye^{2xy} \rightarrow$$

$$\rightarrow \quad b(1 + 2xy) = 1 + 2xy \rightarrow \quad b = 1.$$

So, the exact equation is the following:

$$(ye^{2xy} + x) + xe^{2xy}\frac{dy}{dx} = 0$$

Solution of this exact equation is $\varphi(x, y) = C$**,**

where $\frac{\partial\varphi}{\partial x} = ye^{2xy} + x$, $\frac{\partial\varphi}{\partial y} = xe^{2xy}$**,** C **is an arbitrary real constant.**

Thus, $\varphi(x, y) = \int xe^{2xy}dy + h(x) = \frac{e^{2xy}}{2} + h(x)$.

From this formula and the previous one obtain

$$\frac{\partial\varphi}{\partial x} = ye^{2xy} + h'(x) = ye^{2xy} + x \rightarrow \quad h(x) = \frac{x^2}{2},$$

$$\varphi(x, y) = \frac{e^{2xy}}{2} + \frac{x^2}{2}.$$

The solution of the equation is $e^{2xy} + x^2 = C_1, hence \; y = \frac{ln(C_1 - x^2)}{2x}, \; x \neq 0.$