Answer on Question #50592 – Math – Calculus

Question

The area, A cm2 , of a circle increases at a constant rate of 2 cm2/s . If the initial area of A is 1 cm2 , show that the radius of the circle at time t is given by r = sqrt(2t+1/pi)

Solution

It is well-known that the area of a circle at time t is given by

$$A(t) = \pi r^2(t), \quad (1)$$

where $r(t)$ is the radius of the circle at time t.

In our case the initial area is

$$A(0) = 1 \text{ cm}^2$$

By statement of question, the area of the circle increases at a constant rate of 2 cm^2/s, so after 1 second obtain $A(1) = 1 + 2 = 3 \ (\text{cm}^2)$, after 2 seconds $A(2) = 1 + 2 \cdot 2 = 5 \ (\text{cm}^2)$ and so on.

The formula for the area after t seconds is given by

$$A(t) = 1 + 2t \ \ (\text{cm}^2) \quad (2)$$

Then equate expressions for $A(t)$ from (1) and (2), which gives the following equation:

$$1 + 2t = \pi r^2(t) \quad (3)$$

Solve equation (3) for $r(t)$:

$$r(t) = \sqrt{\frac{1 + 2t}{\pi}}$$