Answer on Question #50591 – Math – Calculus

Question

The volume, $V\ (cm^3)$, of a cone height h is

$$V = \pi \frac{h^3}{12}.$$

If h increases at a constant rate of 0.2 cm/sec and the initial height is 2 cm, express V in terms of t and find the rate of change of V at time t.

Solution:

We know that h increases at a constant rate of 0.2 cm/sec, it means that

$$\frac{dh}{dt} = 0.2\ (cm/sec).$$

Integrate both sides of this equality with respect to t in order to find

$$h(t) = \int 0.2dt = 0.2t + c.$$

Because the initial height is

$$h(0) = 2\ (cm),$$

then

$$h(0) = 0.2 \cdot 0 + c = 2,$$

$$c = 2.$$

So

$$\boxed{h(t) = 0.2t + 2 = 0.2(t + 10)}\ \ (cm).$$

Thus, the volume of cone expressed in terms of t is

$$V(t) = \pi \frac{h^3}{12} = \pi \frac{(0.2(t + 10))^3}{12} = \frac{\pi}{1500}(t + 10)^3\ \ (cm^3)\ .$$

And finally the rate of change of V at time t is the derivative of $V(t)$ with respect to t:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi}{1500}(t + 10)^3\right) = \frac{\pi}{1500} \cdot \frac{d}{dt}((t + 10)^3) = \frac{\pi}{1500} \cdot 3(t + 10)^2 =$$

$$= \frac{\pi}{500}(t + 10)^2\ (cm^3/sec).$$

Answer: $V(t) = \frac{\pi}{1500}(t + 10)^3\ \ (cm^3),\ \ \frac{dV}{dt} = \frac{\pi}{500}(t + 10)^2\ (cm^3/sec)$