Question

The radius, r cm, of a spherical balloon at a time t seconds is given by $r = 3 + \frac{2}{1+t}$.

What is the initial radius? Find the rate of change of r (w.r.t. t) when $t = 3$

Solution

The initial radius is evaluated when $t = 0$:

$$r(0) = 3 + \frac{2}{1+0} = 3 + 2 = 5 \ (cm)$$

The rate of change of r (w.r.t. t) is the following:

$$\frac{dr(t)}{dt} = \frac{d}{dt}\left(3 + \frac{2}{1+t}\right) = \frac{d}{dt}(3) + \frac{d}{dt}\left(\frac{2}{1+t}\right) = 0 + 2 \cdot \left(-\frac{1}{(1+t)^2}\right) = -\frac{2}{(1+t)^2}$$

The rate of change of r (w.r.t. t) when $t = 3$ is given by

$$\frac{dr(t)}{dt}\Big|t = 3 = -\frac{2}{(1+3)^2} = -\frac{2}{4^2} = -\frac{2}{16} = -0.125 \ \left(\frac{cm}{s}\right).$$

Answer: $r(0) = 5 \ cm, \frac{dr(t)}{dt}\Big|t = 3 = -0.125 \frac{cm}{s}$