Task:

The radius r (cm) of a circle at time t seconds is given by r = 9t - t^3 . At each of the following instants, find the rate of change of the radius (w.r.t.t) and state whether the radius is increasing or decreasing at these instants.

a) t =1

b) t =2

c) t =2.5

Answer:

The rate of change of the radius is calculated by $\frac{dr}{dt}$ = $9 - 3t^2$ (cm).

So let t_1 =1, t_2 =2, t_3 =2.5.

If $\frac{dr}{dt}(t_n) > 0$, then the radius is increasing at t_n instant.

If $\frac{dr}{dt}(t_n) < 0$, then the radius is decreasing at t_n instant.

Evaluate

$\frac{dr}{dt}(t_1) = \frac{dr}{dt}(1) = 9 - 3 \cdot 1^2 = 9 - 3 = 6 > 0,$

$\frac{dr}{dt}(t_2) = \frac{dr}{dt}(2) = 9 - 3 \cdot 2^2 = -3 < 0,$

$\frac{dr}{dt}(t_3) = \frac{dr}{dt}(2.5) = 9 - 3 \cdot 2.5^2 = 9 - 3(2 + 0.5)^2 = 9 - 3 \cdot 2^2 - 3 \cdot 2 \cdot 2 \cdot 0.5 - 3 \cdot 0.5^2 =$

$= -3 - 6 - 0.75 = -9.75 < 0$

Thus, the radius is increasing at t =1 and decreasing at t =2, t =2.5.