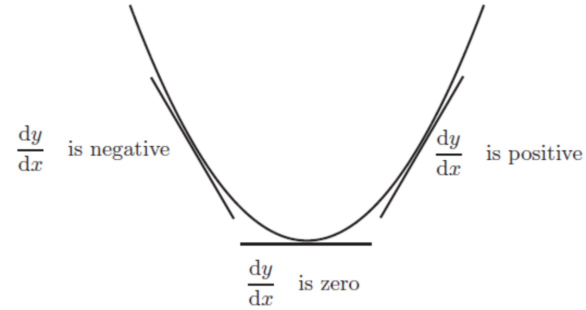
Answer on Question #50562 – Math - Differential Calculus | Equations

Question

Why a minimum is found when second derivative is greater than zero?

Answer

Think about what happens to the gradient of the graph as we travel through the minimum turning point, from left to right, that is, as x increases. Study Figure to help you do this.



$\frac{dy}{dx}$ goes from negative through zero to positive as x increases.

We have

$$\frac{d^2y}{dx^2} = \lim_{h\to 0}\frac{\frac{dy}{dx}(x+h) - \frac{dy}{dx}(x)}{h} = \lim_{h\to 0}\frac{\frac{dy}{dx}(x+h) - 0}{h} = \lim_{h\to 0}\frac{\frac{dy}{dx}(x+h)}{h}.$$

From the figure we see

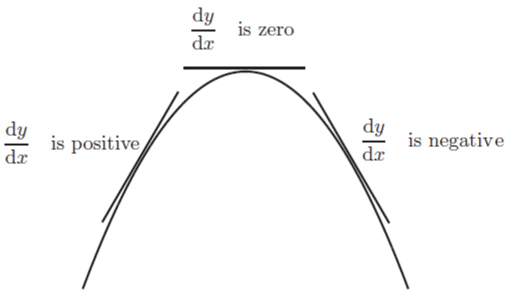
$$\frac{dy}{dx}(x+h) > 0 \; when \; h > 0 \to \frac{d^2y}{dx^2} = \lim_{h\to 0}\frac{\frac{dy}{dx}(x+h)}{h} > 0.$$

Question

Why a maximum is found when second derivative is less than zero?

Answer

Now think about what happens to the gradient of the graph as we travel through the maximum turning point, from left to right, that is as x increases. Study Figure to help you do this.



$\frac{dy}{dx}$ goes from positive through zero to negative as x increases.

We have

$$\frac{d^2y}{dx^2} = \lim_{h\to 0} \frac{\frac{dy}{dx}(x+h) - \frac{dy}{dx}(x)}{h} = \lim_{h\to 0} \frac{\frac{dy}{dx}(x+h) - 0}{h} = \lim_{h\to 0} \frac{\frac{dy}{dx}(x+h)}{h}.$$

From the figure we see

$$\frac{dy}{dx}(x+h) < 0 \; when \; h > 0 \to \frac{d^2y}{dx^2} = \lim_{h\to 0} \frac{\frac{dy}{dx}(x+h)}{h} < 0.$$

Question

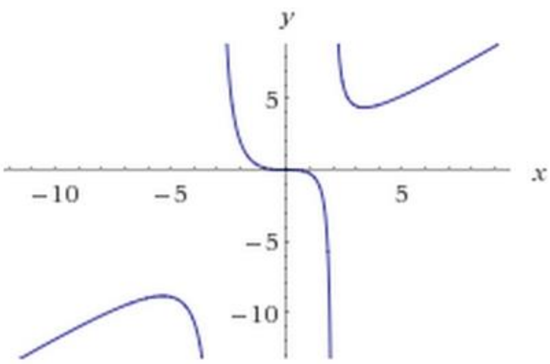
Why and when the minima is greater than maxima. Please explain with example.

Answer

The minima is greater than maxima when they are local maxima and local minima. Global maximum is always greater than global minimum.

Example:

$$y = \frac{x^3}{(x-2)(x+3)}.$$



The local maximum $x_{max} < -3$ and $y(x_{max}) < 0$.

The local minimum $x_{min} > 2$ and $y(x_{min}) > 0$.

In this example $y(x_{max}) < y(x_{min})$, that is, the local minimum is greater than the local maximum.