**Answer on Question #50547 – Math - Differential Calculus | Equations**

sketch the curve x^3-x^2-6x

**Solution.**

$y = x^3 - x^2 - 6x$

1) **Domain:** $-\infty < x < \infty$.

2) **Range:** $-\infty < y < \infty$. **Note that** $y \to \infty$ **as** $x \to \infty$, $y \to -\infty$ **as** $x \to -\infty$

3) **Intercepts:**

$x - intercepts$: $y = 0 \to x^3 - x^2 - 6x = 0 \to x(x^2 - x - 6) = 0 \to x(x-3)(x+2) = 0 \to x = 0, x = 3,\ x = -2, hence\ (0,0), (-2,0), (3,0)\ are\ x - intercepts$

$y - intercept$: $x = 0 \to y(0) = 0^3 - 0^2 - 6 \cdot 0 = 0, hence$

$(0,0)$ **is** $y - intercept.$

4) **Symmetry:** $y(x) is\ neither\ even\ nor\ odd\ function,$

$$because\ y(-x) \neq -y(x), y(-x) \neq y(x)$$

5) **Asymptotes: there are no asymptotes.**

6) **Local extrema:** $y' = 0 \to\ 3x^2 - 2x - 6 = 0 \to\ x = \frac{1-\sqrt{19}}{3},\ or\ x = \frac{1+\sqrt{19}}{3}$.

$\left(\frac{1-\sqrt{19}}{3}, \frac{38\sqrt{19}-56}{27}\right) - local\ maximum;$

$\left(\frac{1+\sqrt{19}}{3}, \frac{-38\sqrt{19}-56}{27}\right) - local\ minimum;$

7) **Intervals of increase and decrease:**

$increasing\ on\ \left(-\infty, \frac{1-\sqrt{19}}{3}\right)\ and\ on\ \left(\frac{1+\sqrt{19}}{3}, \infty\right);$

$$decreasing\ on\ \left(\frac{1-\sqrt{19}}{3}, \frac{1+\sqrt{19}}{3}\right);$$

**8) Concavity and points of inflection:**

$$y''(x) = 0 \rightarrow \ 6x - 2 = 0 \rightarrow \ x = \frac{1}{3};$$

**concave downward on** $\left(-\infty, \frac{1}{3}\right)$**, concave upward on** $\left(\frac{1}{3}, \infty\right)$;

**point of inflection is** $\left(\frac{1}{3}, -\frac{19}{3}\right)$.

**Graph of $y$ as function of $x$ is shown below:**