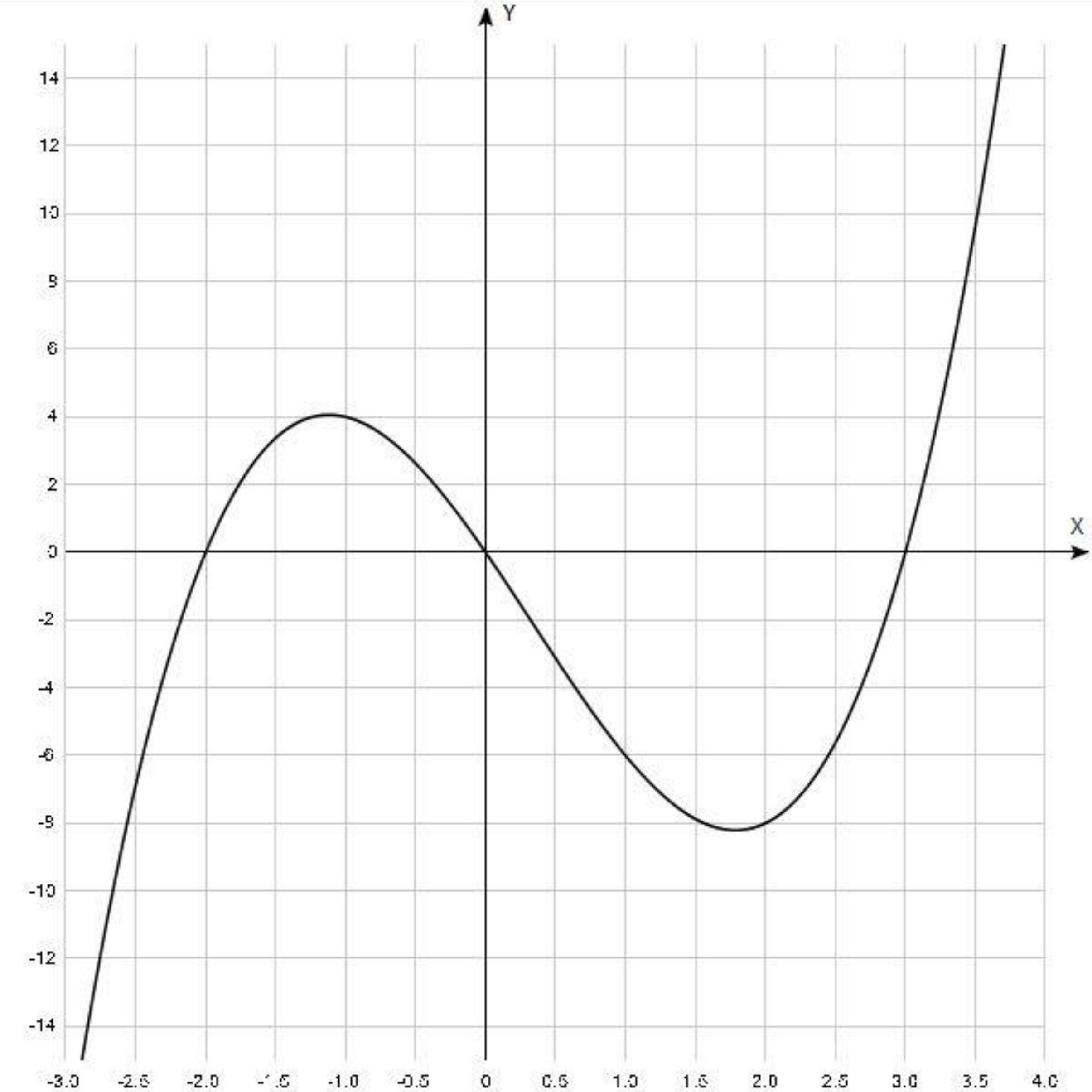
Answer on Question #50544 – Math – Differential Calculus | Equations



The greatest value of y is ∞ or approximately -1.555. See reasons on the last page.

Question

Sketch the curve $x^3 - x^2 - 6x$, if $x - 2y = 6$, find the greatest value of y.

Solution

Let us make some anchor points.

First and second derivative of the function.

$$\frac{d}{dx}(x^3 - x^2 - 6x) = 3x^2 - 2x - 6$$

$$\frac{d^2}{dx^2}(x^3 - x^2 - 6x) = 6x - 2$$

Extreme points.

$$3x^2 - 2x - 6 = 0$$

$$x^2 - \frac{2}{3}x - 2 = 0$$

$$x_1 = \frac{1 - \sqrt{19}}{3}, \quad x_2 = \frac{1 + \sqrt{19}}{3}.$$

x_1: $6x_1 - 2 = 2 - 2\sqrt{19} - 2 = -2\sqrt{19} < 0$ – maximum.

x_2: $6x_2 - 2 = 2 + 2\sqrt{19} - 2 = 2\sqrt{19} > 0$ – minimum.

Corresponding y positions of the points.

y_1: $x_1^3 - x_1^2 - 6x_1 = -\frac{56}{27} + \frac{38\sqrt{19}}{27}$

y_2: $x_2^3 - x_2^2 - 6x_2 = -\frac{56}{27} - \frac{38\sqrt{19}}{27}$

Inflection point.

$$6x - 2 = 0$$

$$x_3 = \frac{1}{3}$$

Corresponding y position of the point.

y_3: $x_3^3 - x_3^2 - 6x_3 = -\frac{56}{27}$

Zeros of the function.

$$x^3 - x^2 - 6x = 0$$

$$x_4 = 0$$

$$x^2 - x - 6 = 0$$

$$x_5 = -2, \quad x_6 = 3.$$

Approximate positions of the points in the order of their appearance.

$$(-1.12; 4.06), \quad (1.79;\ -8.21), \quad (0.33;\ -2.07), \quad (0; 0), \quad (-2; 0), \quad (3;\ 0).$$

Behavior of the function at $-\infty$ and $+\infty$.

$$\lim_{x \to -\infty} x^3 - x^2 - 6x = -\infty$$

$$\lim_{x \to \infty} x^3 - x^2 - 6x = \infty$$

That’s all we need to sketch the curve.

Regarding $x - 2y = 6$ we have two options:

If y is an independent variable then its greatest value going to ∞, because $y = \frac{x}{2} - 3$, domain of x is $\mathbb{R}$.

If $y = x^3 - x^2 - 6x$, then we have equation

$$x - 2x^3 + 2x^2 + 12x = 6 \rightarrow x^3 - x^2 - \frac{13}{2}x + 3 = 0$$

Approximate solutions.

$$x_1 = -2.335, \;\; x_2 = 0.445, \;\; x_3 = 2.890.$$

Corresponding y values.

$y_1: x_1^3 - x_1^2 - 6x_1 \approx -4.173$

$y_2: x_2^3 - x_2^2 - 6x_2 \approx -2.780$

$y_3: x_3^3 - x_3^2 - 6x_3 \approx -1.555$

Hence, the greatest value of $y \approx -1.555$.