Answer on Question #50525– Math – Integral Calculus

$$\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$$

Solution:

Integrating by parts

$$u = sin^{-1}\sqrt{\frac{x}{a+x}}, \qquad du = \frac{1}{\sqrt{1-\frac{x}{a+x}}}\cdot\frac{1}{2}\sqrt{\frac{a+x}{x}}\cdot\frac{a+x-x}{(a+x)^2}dx = \frac{\sqrt{a}}{2\sqrt{x}(a+x)}dx$$

$$dv = dx, v = x$$

$$\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx = uv - \int vdu = x\sin^{-1}\sqrt{\frac{x}{a+x}} - \int \frac{\sqrt{a}x}{2\sqrt{x}(a+x)}dx$$

Let $\sqrt{x} = t$, then $x = t^2$, and $\frac{dx}{2\sqrt{x}} = dt$, so $dx = 2\sqrt{x}dt = 2tdt$

Hence:

$$\int \frac{\sqrt{a}x}{2\sqrt{x}(a+x)}dx = \int \frac{\sqrt{a}t}{2(a+t^2)}\cdot 2tdt = \sqrt{a}\int \frac{t^2+a-a}{t^2+a}dt =$$

$$= \sqrt{a}\int dt - a\sqrt{a}\int \frac{dt}{t^2+a} = \sqrt{a}t - \frac{a\sqrt{a}}{\sqrt{a}}\tan^{-1}\frac{t}{\sqrt{a}} + C =$$

$$= \sqrt{ax} - a\tan^{-1}\frac{\sqrt{x}}{\sqrt{a}} + C,$$

where C is an arbitrary real constant.

Finally obtain:

$$\boldsymbol{\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx = x\sin^{-1}\sqrt{\frac{x}{a+x}} - \sqrt{ax} + a\tan^{-1}\frac{\sqrt{x}}{\sqrt{a}} + C}$$