Answer on Question #50472 – Math – Integral Calculus

Solve ∫((1+∜x)/((1+√x))dx.

Solution

We'll use a substitution $t = \sqrt[4]{x}$ to avoid radicals.

$$\int \frac{1+\sqrt[4]{x}}{1+\sqrt{x}}dx = \int \frac{1+\sqrt[4]{x}}{1+\left(\sqrt[4]{x}\right)^2}dx = \left| \begin{matrix} t = \sqrt[4]{x} \\ x = t^4 \\ dx = 4t^3dt \end{matrix} \right| = \int \frac{1+t}{1+t^2} \cdot 4t^3dt = 4\int \frac{t^4+t^3}{t^2+1}dt$$

$$= 4\int \frac{t^2(t^2+1)+t(t^2+1)-(t^2+1)-t+1}{t^2+1}dt$$

$$= 4\int \left(t^2+t-1-\frac{t}{t^2+1}+\frac{1}{t^2+1}\right)dt$$

$$= 4\int t^2dt + 4\int tdt - 4\int dt - 4\int \frac{t}{t^2+1}dt + 4\int \frac{1}{t^2+1}dt$$

$$= 4\cdot\frac{t^3}{3} + 4\cdot\frac{t^2}{2} - 4\cdot t - 4\int \frac{t}{t^2+1}dt + 4\tan^{-1}t + C$$

$$= \frac{4}{3}t^3 + 2t^2 - 4t + 4\tan^{-1}t + C - 2\int \frac{2tdt}{t^2+1}$$

$$= \frac{4}{3}t^3 + 2t^2 - 4t + 4\tan^{-1}t + C - 2\int \frac{d(t^2+1)}{t^2+1} = |\text{substitution } u = t^2+1|$$

$$= \frac{4}{3}t^3 + 2t^2 - 4t + 4\tan^{-1}t + C - 2\int \frac{du}{u}$$

$$= \frac{4}{3}t^3 + 2t^2 - 4t + 4\tan^{-1}t - 2\ln|u| + C$$

$$= \frac{4}{3}t^3 + 2t^2 - 4t + 4\tan^{-1}t - 2\ln|t^2+1| + C$$

$$= \frac{4}{3}\sqrt[4]{x^3} + 2\sqrt{x} - 4\sqrt[4]{x} + 4\tan^{-1}\sqrt[4]{x} - 2\ln\left(\sqrt{x}+1\right) + C$$

where C is an arbitrary real constant, the sign | | of absolute value is omitted, because expression $\sqrt{x}+1$ is positive.

We used the next table integrals: $\int y^n dy = \frac{y^{n+1}}{n+1} + C, n \neq -1, \int \frac{dy}{y} = ln|y| + C,$

$\int \frac{dy}{1+y^2} = tan^{-1}(y) + C$, where C is an arbitrary real constant, $tan^{-1}(y)$ means the inverse tangent function (it is sometimes called $\arctan(y)$).

Answer:

$$\int \frac{1+\sqrt[4]{x}}{1+\sqrt{x}}dx = \frac{4}{3}\sqrt[4]{x^3} + 2\sqrt{x} - 4\sqrt[4]{x} + 4\tan^{-1}\sqrt[4]{x} - 2\ln\left(\sqrt{x}+1\right) + C$$