Answer on Question #50469 – Math – Calculus

The maximum value of [x(x-1) +1]1/3 , 0 ≤ x ≤ 1 is ?

Solution

Function $f(x)=\frac{1}{3}(x(x-1)+1)$ is continuous on the segment $[0;1]$ and differentiable at all points of this segment except, possibly, finitely many points. Then the largest and the smallest values of $f(x)$ on $[0;1]$ belong to the set consisting of $f(0)$, $f(1)$, and the values $f(x_i)$, where $x_i \in (0;1)$ are the points at which $f'(x)$ is either equal to zero or does not exist (is infinite).

Evaluate $f(0)=\frac{1}{3}(0(0-1)+1)=\frac{1}{3}(0+1)=\frac{1}{3}$,

$f(1)=\frac{1}{3}(1(1-1)+1)=\frac{1}{3}(0+1)=\frac{1}{3}$, i.e. $f(0)=f(1)=\frac{1}{3}$.

$$f'(x)=\left(\frac{1}{3}(x(x-1)+1)\right)'=|additive\ property|=\frac{1}{3}\big(x(x-1)\big)'+\frac{1}{3}(1)'=\frac{1}{3}\big(x(x-1)\big)'+0$$
$$=\frac{1}{3}\big(x(x-1)\big)'=|the\ derivative\ of\ the\ product|=\frac{1}{3}(x)'(x-1)+\frac{1}{3}x(x-1)'$$
$$=\frac{1}{3}(x-1)+\frac{1}{3}x=\frac{2}{3}x-\frac{1}{3}=\frac{2x-1}{3}=0,$$

hence $2x-1=0$, so $x=\frac{1}{2}$.

Calculate $f\left(\frac{1}{2}\right)=\frac{1}{3}\left(\frac{1}{2}\left(\frac{1}{2}-1\right)+1\right)=\frac{1}{3}\left(-\frac{1}{2}\cdot\frac{1}{2}+1\right)=\frac{1}{3}\left(-\frac{1}{4}+1\right)=\frac{1}{3}\cdot\frac{3}{4}=\frac{1}{4}<\frac{1}{3}$.

Thus, the maximum of $\frac{1}{4}$ and $\frac{1}{3}$ is $\frac{1}{3}$, that is why the maximum value of $f(x)=\frac{1}{3}(x(x-1)+1), 0\le x\le 1$, is $\frac{1}{3}$, because $f(0)=f(1)=\frac{1}{3}$.