

Answer to Question #50467

$$\begin{aligned}\int \frac{\sqrt{x}}{1 + \sqrt[3]{x}} dx &= \left\{ \text{substitute } u = \sqrt[6]{x} \text{ and } du = \frac{dx}{6x^{\frac{5}{6}}} \right\} \\&= 6 \int \frac{u^8}{1 + u^2} du \\&= 6 \int \left(u^6 - u^4 + u^2 + \frac{1}{u^2 + 1} - 1 \right) du \\&= 6 \left(\int \frac{1}{u^2 + 1} du + \int u^6 du - \int u^4 du + \int u^2 du - \int 1 \cdot du \right) \\&= 6 \tan^{-1} u + \frac{6u^7}{7} - \frac{6u^5}{5} + 2u^2 - 6u = \left\{ \text{substitute back } u = \sqrt[6]{x} \right\} \\&= \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \tan^{-1}(\sqrt[6]{x}) + C\end{aligned}$$

Answer:

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \tan^{-1}(\sqrt[6]{x}) + C$$