

Answer on Question #50441 - Math - Abstract Algebra

Let (X, f_1, \dots, f_n) and (Y, g_1, \dots, g_n) are two algebraic structures, it means that f_i, g_i are maps from X^{n_i} to X and from Y^{m_i} to Y respectively, where m_i is an arity of f_i, g_i .

Then the map $f : X \rightarrow Y$ is called an isomorphism if for every $i \leq n$ and for every $x_1, \dots, x_{m_i} \in X$

$$f(f_i(x_1, \dots, x_{m_i})) = g_i(f(x_1), \dots, f(x_{m_i}))$$

For example rings have two operations $+, \times$, so the map $f : X \rightarrow Y$ is an isomorphism between two rings $(X, +, \times)$ and $(Y, +, \times)$ if for every $x_1, x_2 \in X$:

$$f(x_1 + x_2) = f(x_1) + f(x_2) \text{ and } f(x_1 \times x_2) = f(x_1) \times f(x_2).$$